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A COMPUTER ANALYSIS OF A TRI-MOORED BUOY STRUCTURE WITH AN INTE--ETC(U)  
MAY 72 S C PAHUJA, R W CORELL  
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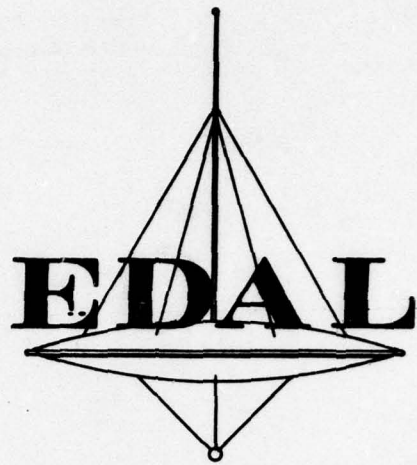






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**A Computer Analysis  
Of a Tri-Moored Buoy Structure  
With an Internally Redundant Horizontal Structural Element**

**S. C. Pahuja  
R. W. Corell**

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Report to the Office of Naval Research

Technical Report 110

May 1972

ENGINEERING DESIGN AND ANALYSIS LABORATORY  
UNIVERSITY OF NEW HAMPSHIRE, DURHAM, NEW HAMPSHIRE

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ENGINEERING DESIGN AND ANALYSIS LABORATORY

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Durham, New Hampshire

⑥ A Computer Analysis

Of a Tri-Moored Buoy Structure With  
An Internally Redundant Horizontal Structural Element

⑨ Technical Report No. 110

By

⑩ S. C. Pahuja

R. W. Corell

⑭ TR-

⑪ May 72

⑫ 89 p.

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The research reported herein was sponsored by the Office of Naval Research, Contract No. ⑮ N00014-67-A-0158-0004

May 1972

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### ACKNOWLEDGEMENTS

The authors wish to express appreciation to G. H. Savage for his thoughtful assistance from time to time and to Dr. Richard A. Skop of Naval Research Laboratories for his constructive comments. Discussions with Dr. Robert Archer of the University of Massachusetts were helpful and are gratefully acknowledged.

Special thanks to Mr. William M. Conroy, Jr., of UNH Computation Center and Mr. Derek S. Terreson of Graphic Arts Center for their help. The assistance of the Computation Centers of the University of New Hampshire, Stanford University and Massachusetts Institute of Technology, where most of the computational work was done, is acknowledged.

The support of the Office of Naval Research (through Contract No. N00014-67-A-0158-0004) and that of the University of New Hampshire is gratefully appreciated.

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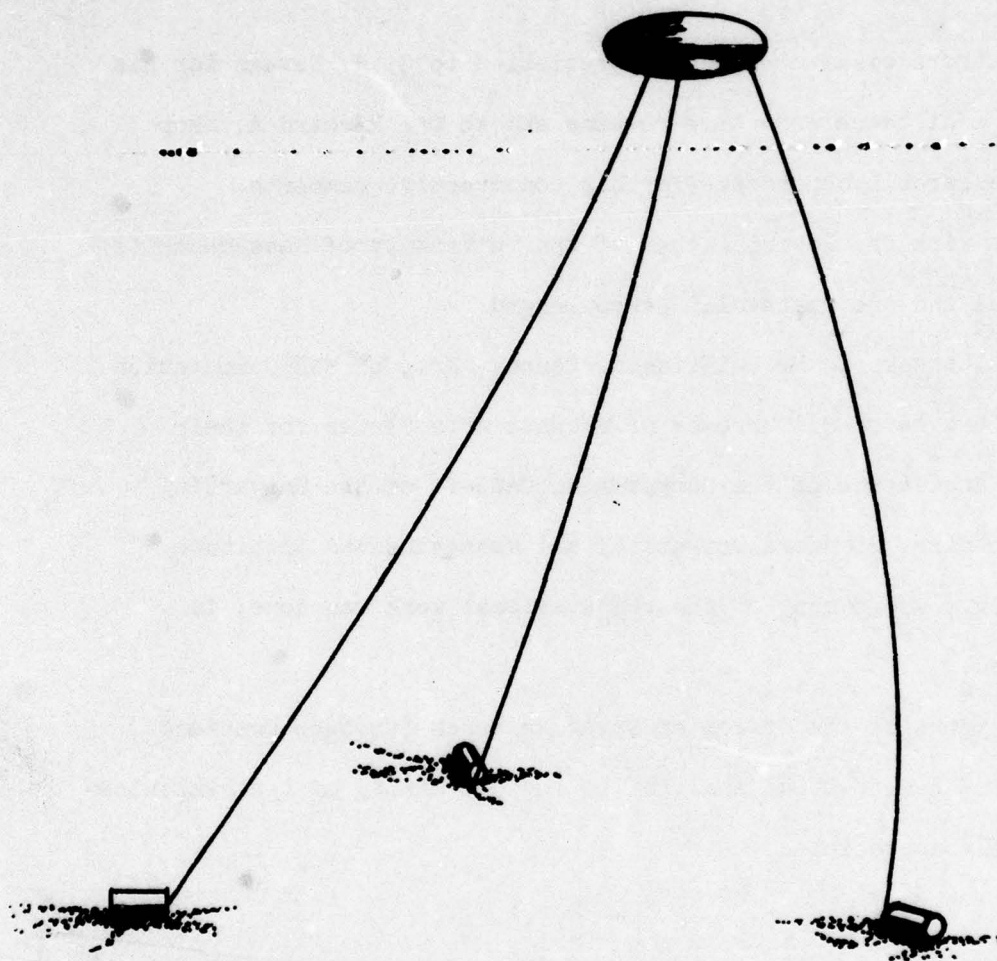


Figure 1: Tri-Moored Subsurface Float with Neutrally Buoyant Legs

# ABSTRACT

↓ This report is concerned with the analysis and computer simulation of a tri-moored oceanic buoy structure having a near-horizontal cable connected between two of the three main (see Figure 1) legs. The Method of Imaginary Reactions is used in conjunction with the Method of Successive Approximations to determine the equilibrium configuration of the cable array. A special technique is developed to extend the above methods to include cable arrays with one internally redundant loop. Computer search routines are developed to ensure convergence to the solution for equilibrium positions of the cable array.

The analysis permits the inclusion of discrete elements and floating devices distributed along all cables. Expressions are derived in detail for the hydrodynamic forces. Both tangential and normal drags are included in the calculation of these forces; however, the forces induced by wave action are not considered.

A computer program for implementing the analysis is included, and a sample output appears in Appendix I. ↗

The work reported herein was conducted during the period of September 1968 through December 1970.

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## SYMBOLS AND NOTATIONS

The symbols and notations used in this report are defined as they appear in the context. The most important ones are listed here for reference:

### A. Symbols as used for Tri-Moored Structure

$(a_n, b_n, c_n)$	the coordinates of the nth cable anchor
$A_{k, m, n}$	the effective cross-sectional area of the (k,m,n)th elemental device
$XTEN(m, n)$	the extensional rigidity of the (m,n,)th cable segment
$C_{m, n}^{c; x}, C_{m, n}^{c; y}, C_{m, n}^{c; z}$	the drag constants of the (m,n)th cable segment
$C_{k, m, n}^{e; x}, C_{k, m, n}^{e; y}$	the drag constants of the (k,m,n)th elemental device
$C_{k, m, n}^D$	the coefficient of drag of the (k,m,n)th elemental device
$C_{m, n}^N$	the coefficient of drag of the (m,n)th cable segment when this segment is normal to the stream
$C_{m, n}^P$	the coefficient of drag of the (M,N)th cable segment when this segment is parallel to the stream
$d_{m, n}$	the diameter of the (m,n)th cable segment
$f_{m, n}^n$	the component of drag force per unit length in the (m,n) direction
$(F_{m, n}^x, F_{m, n}^y, F_{m, n}^z)$	the components of the external force acting at the (m,n)th cable station

$(F_{M(n),n}^x, F_{M(n),n}^y, F_{M(n),n}^z)$

the components of the imaginary reactions applied to the  $M(n)$ th stations of cables 2 and 3

$(h_{m,n}^{c;x}, h_{m,n}^{c;y}, h_{m,n}^{c;z})$

the components of the hydrodynamic force per unit length acting on the  $(m,n)$ th cable segment

$(h_{k,m,n}^{e;x}, h_{k,m,n}^{e;y}, h_{k,m,n}^{e;z})$

the components of the hydrodynamic force acting on the  $(k,m,n)$ th elemental device

$(H_{m,n}^{c;x}, H_{m,n}^{c;y}, H_{m,n}^{c;z})$

the components of the lumped drag force at the  $(m,n)$ th cable station due to the distributed hydrodynamic forces along the cables

$(H_{m,n}^{e;x}, H_{m,n}^{e;y})$

the components of the lumped drag force at the  $(m,n)$ th cable station due to the hydrodynamic forces on the elemental devices

$(\vec{i}, \vec{j}, \vec{k})$

unit vectors in the  $(x,y,z)$  directions, respectively

$(k, m, n)$

the index of the  $K$ th elemental device on the  $m$ th segment of the  $n$ th cable

$\bar{k}(m,n)$

the total number of elemental devices attached to the  $(m,n)$ th cable half-segment adjoining the  $(m-1,n)$ th cable station

$K(m,n)$

the total number of elemental devices attached to the  $(m,n)$ th cable segment

$BL_{m,n}$

the stressed length of the  $(m,n)$ th cable segment

$BL_{0m,n}$

the unstressed length of the  $(m,n)$ th cable segment

$(m,n)$

the index of the  $m$ th station or segment on the  $n$ th cable

$M(n)$

the total number of stations or segments on the  $n$ th cable

$r_{m,n}^D$

the ratio of drag coefficients  
for the (m,n)th cable segment =

$$C_{m,n}^P / C_{m,n}^N$$

$(R_{m,n}^x, R_{m,n}^y, R_{m,n}^z)$

the components of the resultant  
force in the (m,n)th cable segment

$s_{k,m,n}$

the stressed distance of the  
(k,m,n)th elemental device from  
the (m-1,n)th cable station

$\bar{s}_{k,m,n}$

the unstressed distance of the  
(k,m,n)th elemental device from  
the (m-1,n)th station

$T_{m,n}$

the tension in the (m,n)th  
cable segment

$V(m,n : \Xi_1, \Xi_2)$

the integral of  $V^2(z)$  along the  
(m,n)th cable segment from  $\Xi_1$   
to  $\Xi_2$  equal to

$$\int_{\Xi_1}^{\Xi_2} V^2 [Z(m,n;\xi)] d\xi$$

$w_{m,n}^c$

the weight (or buoyancy) per  
unit length in water of the  
(m,n)th cable segment

$w_{k,m,n}^e$

the weight (or buoyancy) in  
water of the (k,m,n)th elemen-  
tal device

$W_{m,n}$

the lumped weight (or buoyancy)  
force at the (m,n)th cable station

$(x,y,z)$

fixed Cartesian coordinates

$(x_{m,n}, y_{m,n}, z_{m,n})$

the coordinates of the (m,n)th  
cable station

$z(m,n;\xi)$

the parametric representation  
of  $z$  along the (m,n)th cable  
segment =  $z_{m-1,n} + \gamma_{m,n}\xi$

$(\alpha_{m,n}, \beta_{m,n}, \gamma_{m,n})$

the direction cosines of the  
(m,n)th cable segment

$\nabla_{m,n}$

the sine of the angle between  
the (m,n)th cable segment and  
the stream



$(\Delta F_{M(n),n}^x, \Delta F_{M(n),n}^y, \Delta F_{M(n),n}^z)$

the components of the additive forces applied to the M(n)th stations of cables 2 and 3

$\mu_{m,n}^c$

the hydrodynamic constant of the (m,n)th cable segment =  $N_{PCm,n} d_{m,n}/2$

$\mu_{k,m,n}^e$

the hydrodynamic constant of the (k,m,n)th elemental device =  $PC_{k,m,n}^D A_{k,m,n}/2$

## B. Symbols As Used for Tie Leg

$A_{k,m}$

the effective cross-sectional area of the (k,m)th elemental device

XXTEN(m)

the extensional rigidity of the m th cable segment

$C_m^{c:x}, C_m^{c:y}, C_m^{c:z}$

the drag constants of the m th cable

$C_m^{e:x}, C_m^{e:y}, C_m^{e:z}$

the drag constants of the (k,m)th elemental device

$C_{j,m}$

the coefficient of drag of the (k,m)th elemental device

$C_m^N$

the coefficient of drag of the m th cable segment when this segment is normal to the stream

$C_m^D$

the coefficient of drag of the m th cable segment when this segment is parallel to the stream

$d_m$

the diameter of the m th cable segment

f

the component of drag force per unit length in the m direction

$(F_m^x, F_m^y, F_m^z)$

the components of the external force acting at the m th cable station

$(h_m^{c:x}, h_m^{c:y}, h_m^{c:z})$

the components of the hydrodynamic force per unit length acting on the mth cable segment

$(h_m^{e:x}, h_m^{e:y}, h_m^{e:z})$

the components of the hydrodynamic force acting on the (k,m)th elemental device

$(H_m^{c:x}, H_m^{c:y}, H_m^{c:z})$

the components of the lumped drag force at the m th cable station due to the distributed hydrodynamic forces along the cables

$(H_m^{e:x}, H_m^{e:y}, H_m^{e:z})$

the components of the lumped drag force at the mth cable station due to the hydrodynamic forces on the elemental devices

$\hat{i}, \hat{j}, \hat{k}$

unit vectors in the (x,y,z) directions, respectively

$(j,m)$

the index of the jth elemental device on the mth segment

$\bar{j}(m)$

the total number of elemental devices attached to the mth cable half-segment adjoining the (m-1)th cable station

$j(m)$

the total number of elemental devices attached to the mth cable segment

$B(m)$

the stressed length of the mth cable segment

$L(M)$

the unstressed length of the m th cable segment

$(m)$

the index of the mth station

$r_m^D$

the ratio of drag coefficients for the mth cable segment =  $c_m^D / c_m$

$R_m^W, R_m^Y, R_m^Z$

the components of the resultant force in the mth cable segment

$S_{k,m}$

the stressed distance of the (k,m)th elemental device from the (m-1) cable station

$\bar{S}_{k,m}$

the unstressed distance of the (k,m)th elemental device from the (m-1)th station

$ST_m$

$V(m; \Xi_1, \Xi_2)$

$W_m^c$

$W_{k,m}^e$

$W_m$

$(x, y, z)$

$(X_m, Y_m, Z_m)$

$Z(m, \xi)$

$(\alpha_m, \beta_m, \gamma_m)$

$\Delta_m$

$\mu_m^c$

$\mu_{k,m}^e$

$\xi$

the tension in the mth cable segment

the integral of  $V^2(z)$  along the mth cable segment from  $\Xi_1$  to  $\Xi_2$

$$\int_{\Xi_1}^{\Xi_2} V^2[Z(m; \xi)] d\xi$$

the weight (or buoyancy) per unit length in water of the mth cable segment

the weight (or buoyancy) in water of the (k,m)th elemental device

the lumped weight (or buoyancy) force at the mth cable station

fixed Cartesian coordinates

the coordinates of the mth cable station

the parametric representation of  $z$  along the mth cable segment =  $Z_{m-1} + \gamma_m \xi$

the direction cosines of the mth cable segment

the sine of the angle between the mth cable segment and the stream

the hydrodynamic constant of the mth cable segment =  $PC_m^N d_{m/2}$

the hydrodynamic constant of the (k,m)th elemental device =  $P_{C,k,m}^D A_{k,m/2}$

a parameter defining distance along the mth cable segment



$(\Gamma_m, \pi_m, \eta_m)$

respectively, unit vectors along the mth cable segment, normal to both the mth cable segment and the stream, and normal to the mth cable segment but in the plane that includes this segment and the stream

C. Symbols that are Common to Both the Main and the Tie Leg Arrays

COMP<sub>D</sub>

a cut off value that defines the acceptable completion of the successive approximation iteration

COMPE

a cut off value that defines the acceptable completion of the Imaginary Reaction iteration

E

a positive definite error function

$\vec{V}$

the current vector

V(z)

the current magnitude at a height z above the bottom

$\delta$

a positive convergence factor having the dimensions of force

$\rho$

the density of surrounding fluid

$\emptyset$

the angular dir<sup>n</sup> of the current with respect to x axis

## INTRODUCTION

The problem of determining undersea cable configurations due to applied loadings at known positions along the cable has received considerable attention in the past (2,5,8,9)\*. However, the solutions have pertained generally to particular loading conditions. A general closed-form analytical method solving a large variety of complex cable systems has not been available, primarily because of the nonlinear characteristic of the differential equations describing these systems.

Alekseev<sup>(1)</sup> dealt with the problem of a free-ended cable from a continuum point of view. He obtained a three-dimensional solution to the equilibrium equations, which included gravity effects, along with arbitrarily applied forces along the cable. Pode<sup>(6)</sup> also used the continuum approach to deal with the above problem. However, in treating a towed body, he only considered a special case of the general problem. Both Alekseev and Pode assumed an inextensible cable so that the exact integration of the equilibrium equations could be obtained. Therefore, these solutions are independent of the materials used for the towing cables.

Walton and Polachek<sup>(10)</sup> approached the problem of a free-ended cable from a lumped parameter point of view. They developed a two-dimensional numerical solution for an inelastic line in water when the end conditions are assumed. Paquette and Henderson<sup>(5)</sup> followed a procedure similar to that of Walton and Polachek using analog computer techniques. The cable was

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\*Numbers within parentheses refer to references given on page 73.



considered to be elastic but constraints were placed on the equilibrium position of the cable stations. Both steel and nylon cables were considered. O'Brien<sup>(2)</sup> considered the case of fixed end elastic cables, such as transmission lines. An exact continuum solution to the problem was obtained; however, the forces applied to the cable were assumed to be constant over sections of prescribed length so that the shape of the cable segments could be expressed as an elastic catenary.

Skop and O'Hara<sup>(8)</sup> developed a technique called the Method of Imaginary Reactions, which is an extension of classical consistent deformation theory to a nonlinear problem. As reported in Reference 8 & 9 the method applies to elastic non-redundant cable systems and uses lumped parameter representations of the external forces. The technique employs a numerical analysis method that uses a set of straight segments to represent the cable. This enables the non-linear system to be represented by a set of linear equations. By prescribing a simple method of varying the redundant reactions, an iteration technique is used which converges to the correct reactions (and consequently the correct static equilibrium configuration). The method is globally convergent and converges to actual reactions from any set of initially estimated reactions. The only restriction reported by the authors (Reference 8 & 9) is that the method is not applicable if internal loops (a redundant structure) exist in the cable system.

Savage and Sniffin<sup>(7)</sup> have analyzed a tri-moored subsurface float with neutrally buoyant mooring legs (see Fig. 1), utilizing a three-dimensional solution that assumes that the neutrally buoyant legs have a catenary shape under the effect of current. They also assume that if the

variation of current velocity with depth is moderate, the use of root mean square velocity as a uniform velocity profile results in good approximation of the cable shapes.

Skop and Kaplan<sup>(9)</sup> applied their method to determining the static configuration of a buoy cable array like the one analyzed by Savage and Sniffin. The cable array is loaded by weights and buoyancy forces and by current-induced hydrodynamic forces which are functions of both the orientation and depth of the cables in water. The Method of Successive Approximations is combined with the Method of Imaginary Reactions to analyze the position dependent forces. Provision is made to allow for varying current profile and a three-dimensional solution is developed.

This analysis uses the Method of Imaginary Reactions to simulate and analyze a structure which is similar to the one dealt with by Skop and Kaplan except for the addition of a near-horizontal cable attached to any two points on the array of the structure. This structure is diagrammatically shown in Figure 2.

Skop and O'Hara<sup>(8)</sup> reported the inapplicability of the Method of Imaginary Reactions to structures that have internal loops, although they have recently published a report which removes this restriction.<sup>(12)</sup> The structure analyzed in this report has one internal loop. Therefore, the purpose of this engineering research is to develop a technique which overcomes the Skop/O'Hara restriction and show that the Method of Imaginary Reactions can be used to analyze a structure with at least one internal loop.

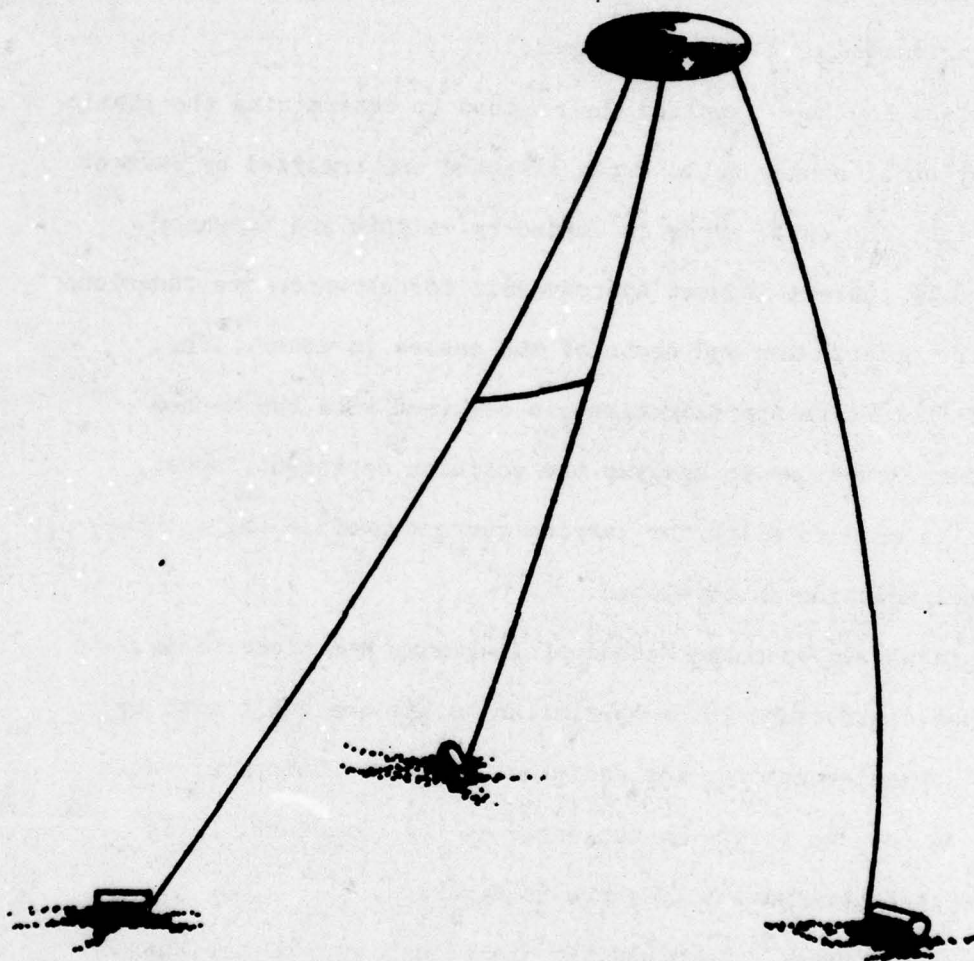


Figure 2: The Tri-Moored Array Structure with a Horizontal  
Element Between Two Cables



Since the work of Pahuja<sup>(11)</sup>, Skop and O'Hara have generalized the Method of Imaginary Reactions to handle the redundant structural cable array.<sup>(12)</sup> This report, summarizing the research work of Pahuja published in 1970, describes the use of the Method of Imaginary Reactions in the analysis of a single loop cable array. The appendix contains the computer program developed which has been compiled on an IBM 360/40 facility.

## CHAPTER I

### METHOD OF IMAGINARY REACTIONS

#### A. Background

A classical method for the analysis of indeterminate structures is the method of consistent deformation, the method of consistent distortion or displacement as developed by James Clark Maxwell.<sup>(4)</sup> This method of indeterminate reaction analysis utilizes equations of compatibility of the structure to supplement the equations of equilibrium to obtain a solution to the unknown redundants. The following assumptions are made in this type of method:

1. The structure is assumed to be linear, i.e., the loads applied are proportional to the displacements and the displacements are relatively small.
2. That there are no gross structural distortions or instabilities upon the release of redundant reactions, i.e., upon the release of the "redundant reaction" the basic geometry of the structure should not change.

The above two conditions are generally not met in the analysis of cable arrays.

The Method of Imaginary Reactions<sup>(8)</sup> overcomes these restrictions for the analysis of cable systems. It is, however, a natural extension of classical consistent deformation theory.

The method uses the following assumptions:

1. The bending stiffness of the cable is neglected.

2. The external forces acting on the cable arrays are "lumped" at stations or nodes along the cable.
3. The cable array is always statically stable, i.e., under the action of applied forces, no cable segment has zero tension.

The first two assumptions are made to help simplify the problem. If the cable length is divided into segments, then each cable segment between stations can be assumed straight. The analysis technique provides the equilibrium configuration of the system (including the effect of cable stretching) which is determined uniquely from formulas that are functions of only the applied forces and the reactions. This offers an advantage, in that the solution is now dependent upon the number of external redundant reactions and not on the number of stations, which are arbitrary. The third assumption is necessary, because even though convergence is still obtained, when this condition is not met the configuration is no longer uniquely described by the method.

## B. Description of the Method of Imaginary Reactions

### 1. Basic Concepts

This method, as adapted to cables by Skop and O'Hara<sup>(8)</sup> is explained briefly in this section. Consider a single cable array of two-dimensional applied forces (the method is not restricted to two dimensions) as shown in Figure 3.



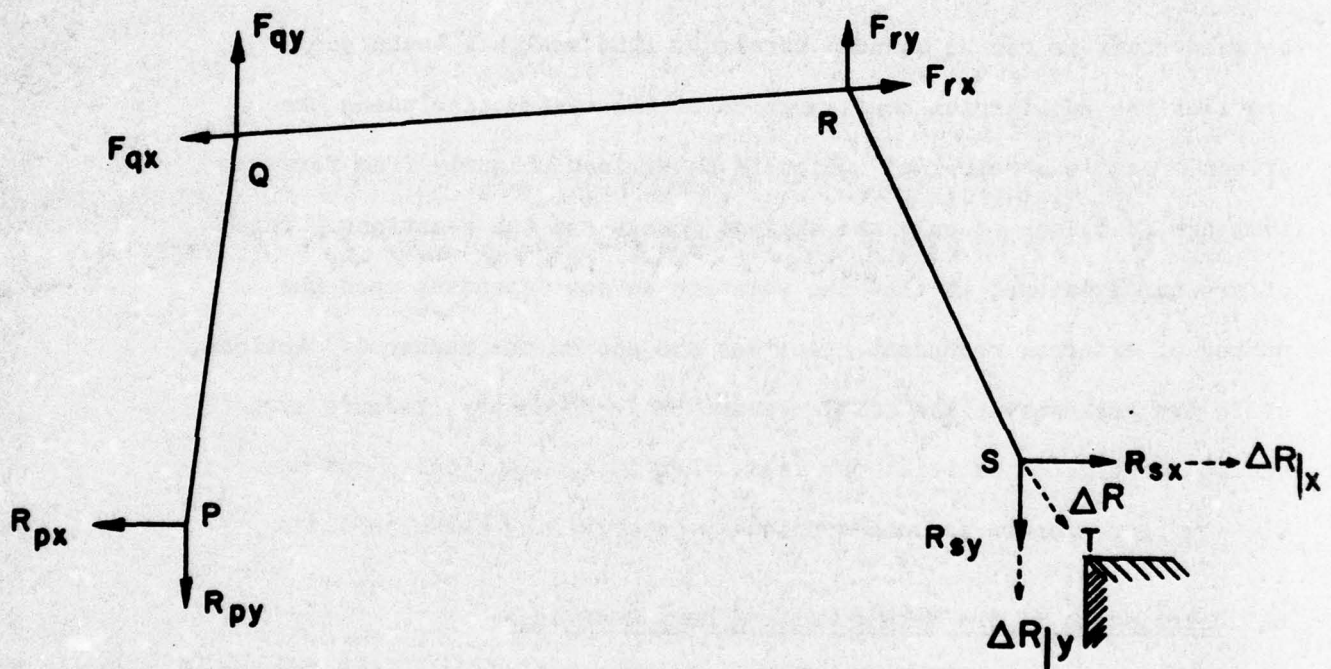


Figure 3: Basic Concepts of the Single Cable

In Figure 3,  $R_{sx}$  and  $R_{sy}$  acting at point S,  $F_{rx}$  and  $F_{ry}$  acting at point R,  $F_{qx}$  and  $F_{qy}$  acting at point Q, are externally applied and are known. If P be designated as the anchor point, then the reaction components  $R_{px}$  and  $R_{py}$  can be determined simply by summation of forces in the x and y directions.

Since these reactions are known, the tension in segment PQ, the length of segment PQ under the tension and the coordinates of point Q become known in that order by the use of elementary statics. Using the same type of analysis the coordinates of point R and S can also be found. Once the position of these points is known, we have obtained the equilibrium configuration of the free-ended cable. If point S happens to be the second anchor point, then the fixed-ended cable system is in equilibrium in the desired configuration. In general, however, we are not at the desired fixed end anchor point, pt. T, but rather at point S. Therefore anchor point reactions  $R_{sx}$  and  $R_{sy}$  at point S are not the actual anchor point reactions which place the cable end at the desired anchor point (T). The Method of Imaginary Reactions provides the methodology for finding the actual reactions at point T which will force point S to move over to the anchor point (T), producing the true equilibrium configuration of the system.

This is done by first applying a small force  $\Delta R$  at point S vectorially directed in a direction from S towards T. If the coordinates of point T are  $(T_x, T_y)$  and those of point S are  $(S_x, S_y)$ , then the ratio of the component of this additional force  $[(\Delta R)_x \text{ \& } (\Delta R)_y]$  is given by:

$$\frac{(\Delta R)_x}{(\Delta R)_y} = \frac{T_x - S_x}{T_y - S_y} \quad (4,1)*$$

---

\*The numbering of all important equations is based upon the format (a,b) where a is the number of an equation in a given chapter b. (Equations (1,1) - (3,1) have been deleted from the text.)



The magnitude of these components is defined by:

$$(\Delta R)_x = \frac{\delta}{\sqrt{E}} (Tx - Sx) \quad (5,1a)$$

$$(\Delta R)_y = \frac{\delta}{\sqrt{E}} (Ty - Sy) \quad (5,1b)$$

where  $E = (Tx - Sx)^2 + (Ty - Sy)^2$  is the measure of error between the equilibrium position calculated using an estimated or guessed reaction and the correct equilibrium position.  $E$  is always positive and vanishes uniquely only when the correct equilibrium has been obtained. In other words,  $E \rightarrow 0$  only when  $Tx = Sx$  and  $Ty = Sy$  simultaneously. However, to arrive at the correct equilibrium configuration, it is necessary to assure that  $E$  will become vanishingly small. To assure convergence, a positive number,  $\delta$ , having the dimensions of the force, is introduced.  $\delta$  is a convergence parameter used to select the magnitude of the additive force  $\Delta R$  (as demonstrated in equations (5,1)) in a manner that the cable array will converge to the correct equilibrium configuration. Referring back to equations (5,1), the ratios  $(Tx - Sx)/\sqrt{E}$  and  $(Ty - Sy)/\sqrt{E}$  are of bounded variation (lying between -1 and +1), and  $E$  will become vanishingly small only when  $\delta \rightarrow 0$ . When this happens, the additive forces  $(\Delta R)_x$  and  $(\Delta R)_y$  are zero, and we have the correct reactions and thus the correct equilibrium configuration.

The algorithm for obtaining a solution is:

1. Make a reasonable engineering guess as to the components of the reaction at the redundant anchor (References 8,9, & 11 discuss methods for making these guesses).
2. Release the redundant anchor, i.e., point S, while maintaining the guessed component reaction forces. This results in the creation of a free-ended cable.

3. Calculate the equilibrium position using static force equilibrium and compute the quantities  $(T_x - S_x)$ ,  $(T_y - S_y)$  and  $E$ .
4. Choose an initial value for  $\delta$  so that the value of  $(\Delta R)_x = \frac{\delta}{\sqrt{E}} (T_x - S_x)$ , and  $(\Delta R)_y = \frac{\delta}{\sqrt{E}} (T_y - S_y)$  can be found.  $\delta$  can be chosen initially to be very large, since it will of necessity become smaller as the solution proceeds step by step.
5. Step 4 results in an additive force equal to  $R' = R + \Delta R$  acting at the assumed free end. Apply this additive force and find the new equilibrium position of the cable.
6. The next step involves comparing the value of  $E'$  (the new measure of error) with  $E$  (the previous measure of error.). Two possible situations can exist:
  - a. If  $E' < E$ , then a successful step has been made because the aim is to reduce the new value of  $E$  to zero. In this case the same value of  $\delta$  is retained and a new value of the additive force  $(\Delta R)$  is calculated and added to the previous  $R$  to calculate again a new equilibrium position. Repeat the process until a stage where  $E' > E$ .
  - b. If  $E < E'$ , then  $\delta$  is too large. The new values that resulted in the new measure of error  $E'$  are rejected. Go back to the last iteration which gave the previous measure of error  $E$ . Then, reduce  $\delta$  by halving its value. Then proceed in a repetitive manner with steps 4 through 6, until the value of  $E$  is arbitrarily small, at which point the released end is arbitrarily close to the real anchor position, and hence, a solution to the problem has been obtained. Figure 4 is a flow diagram describing the procedure.

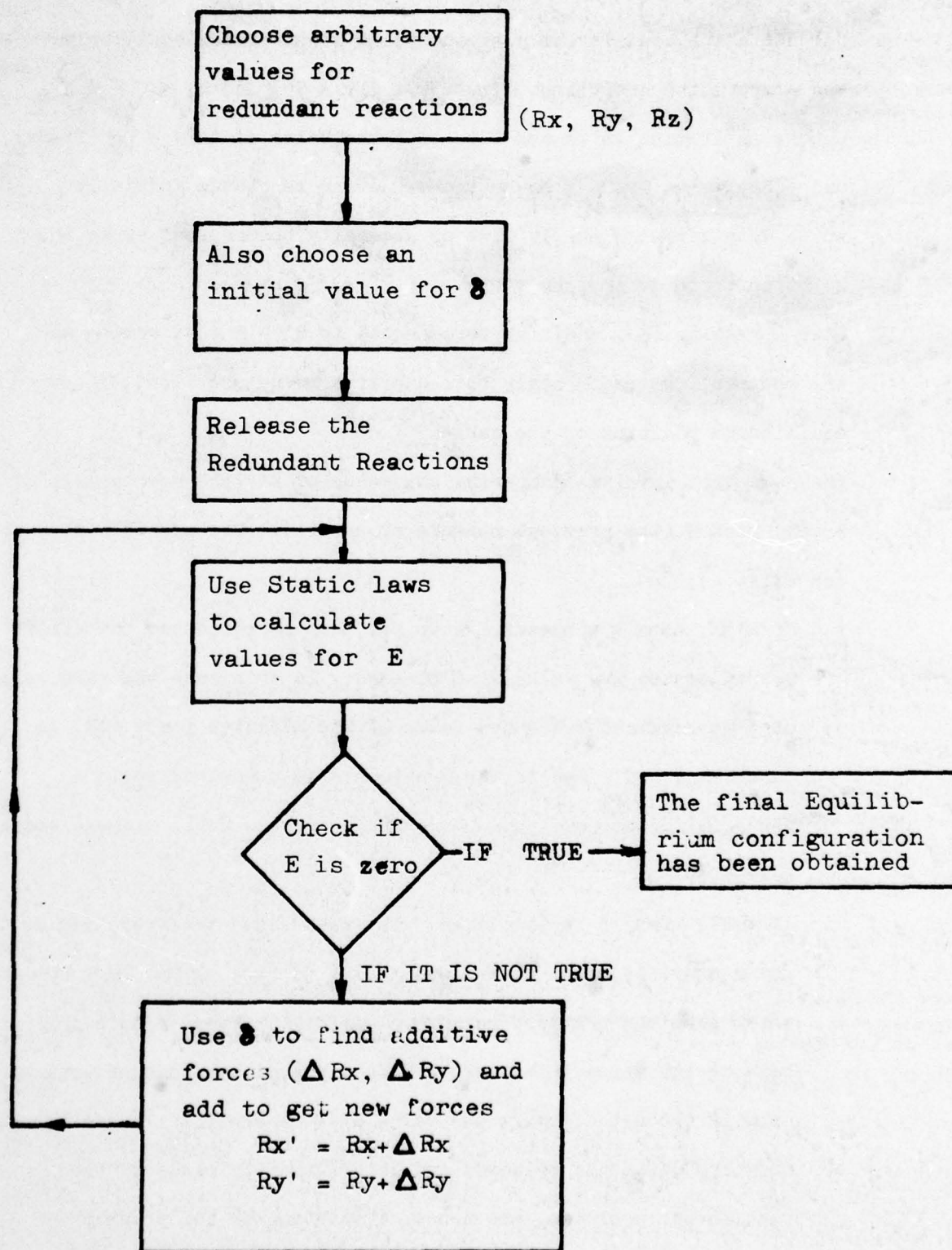


Figure 4: A Flow Diagram Describing the Method of Imaginary Reactions



This application of Imaginary Reactions is globally convergent <sup>(8)</sup>; i.e., no matter what the initial guessed reactions are, the calculation converges to the correct coordinates\*.

## 2. LUMPED PARAMETER REPRESENTATION

One of the assumptions made during the development of the Method of Imaginary Reactions is that the distributed forces that act on the cable array can be represented by lumped forces at specified points on the cable. This assumption is required if the paths to the free ends of the system are to be determined by elementary statics and formulas that express the elongation of the cables under tension. The only condition that these elongation formulas have to meet is that the cable length increases with the increase in tension and be a single valued function; therefore, the elongation formulas can be either linear or non-linear.

Some of the guidelines for a successful lumped parameter representation are presented below:

1. Each cable in the array is represented by at least two stations, one at each end.
2. Each point of discontinuity in a physical property of a cable is represented by a station. Consequently, each cable segment in the array is homogeneous with constant characteristics. The cable can be described by as many stations (segments) as are necessary to represent the cable. Segments into which the cable is divided do not have to be of equal length along the cable array.
3. The method of analysis has no direct bearing on the number of stations on the cable array. As many additional stations as

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\*The analytical proof can be found in Reference 8.

are necessary to obtain a satisfactory approximation to the continuous equilibrium shape of the array are used. The error function  $E$ , the positive  $\delta$ , and the various values of  $\Delta R$  do not depend upon the number of stations.

4. The method by which the external forces acting on the array are to be lumped at the cable station is completely arbitrary. In this report a half segment technique suggested by Paquette and Henderson<sup>(5)</sup> is used.

#### C. APPLICATION OF THE METHOD OF IMAGINARY REACTIONS TO DETERMINE EQUILIBRIUM CONFIGURATION OF A MULTI-MOORED CABLE SYSTEM

In this section, the Method of Imaginary Reactions is discussed in summary fashion to determine the static equilibrium configuration of a multi-moored cable system. (See Figure 1) A detailed discussion of the analysis of this structure is contained in Reference 9 & 11. A summary discussion of the application of the Skop & O'Hara method follows as background to the extension to the single internal loop analysis.

##### 1. Definitions

A few definitions, which are helpful in understanding the further text, are listed below.

##### 1. Simple cable array

A simple cable array is defined as a cable system which contains no closed loops, i.e., it is not internally redundant.

##### 2. A branch point

A branch point of a simple cable array is defined as a point at which a single cable "splits" into two or more cables.

##### 3. Primary and Secondary Anchors

Each cable, in any cable system, is denoted by a number (n). The cable is said to terminate at a branch point. The

index  $n=1$  is reserved for the cable that is attached to an anchor point in the system. In case more than one anchor point exists in a system, then the point to which cable number 1 is attached is called the Primary Anchor and the other anchors are called Secondary Anchors. It is the Secondary Anchors that are released and subjected to imaginary reactions.

## 2. Coordinate System and Notation

For the tri-moored system analysis, a right-handed  $(x,y,z)$  cartesian coordinate system is used. The  $z$  axis is defined to be parallel to the direction of gravity.

The location of the  $n$ th cable anchor is given by  $a(n)$ ,  $b(n)$ ,  $c(n)$ . Each cable in the system is represented by  $M(n)$  stations. The location of the  $m$ th station on the  $n$ th cable is denoted by

$$(X_{m,n}, Y_{m,n}, Z_{m,n}) \quad (21,1)$$

where  $m = 1, 2, \dots, M(n)$ .

The stations are counted from the primary anchor to the branch point of the array along cable 1, and from branch point of the array to the secondary anchors along cables 2 and 3 respectively as shown in Figure 5.

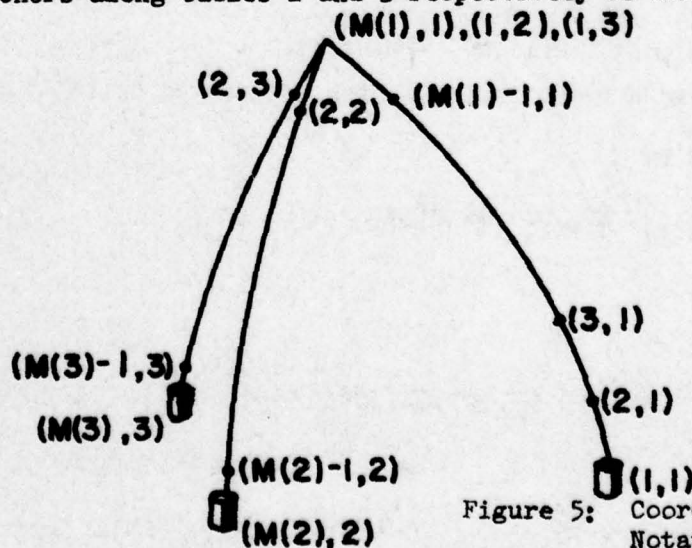


Figure 5: Coordinate Systems and Notations for Tri-Moored Array



Thus, for a branch point

$$X_{(1,2)} = X_{(1,3)} = X_{(M(1),1)} \quad (22,1a)$$

$$Y_{(1,2)} = Y_{(2,3)} = Y_{(M(1),1)} \quad (22,1b)$$

$$Z_{(1,2)} = Z_{(2,3)} = Z_{(M(1),1)} \quad (22,1c)$$

and coordinates of the three anchor points, both primary and secondary are denoted by

$$(X_{(1,1)}, Y_{(1,1)}, Z_{(1,1)}) = (a, b, c) \quad (23,1a)$$

$$(X_{(M(2),2)}, Y_{(M(2),2)}, Z_{(M(2),2)}) = (a_2, b_2, c_2) \quad (23,1b)$$

$$(X_{(M(3),3)}, Y_{(M(3),3)}, Z_{(M(3),3)}) = (a_3, b_3, c_3) \quad (23,1c)$$

The external forces acting at the  $(m,n)$  th station are defined by the components along the  $x, y$  and  $z$  axis:

$$(F_{(m,n)}^x, F_{(m,n)}^y, F_{(m,n)}^z) \quad (24,1)$$

where  $m = 1, 2 \dots M(n)$

and  $n = 1, 2, 3$

Thus, forces acting at an anchor point are represented by

$$(F_{(M(n),n)}^x, F_{(M(n),n)}^y, F_{(M(n),n)}^z) \quad (25,1)$$

and forces at a branch point, which is indexed by  $(M(1),1)$  conventionally, are denoted by

$$F_{(M(1),1)}^x, F_{(M(1),1)}^y, F_{(M(1),1)}^z \quad (26,1)$$

### 3. Static Equilibrium Configuration

It was stated earlier that for a two-dimensional cable system if external forces in segments are known, then the tensions in and orientation of the cable segments can be uniquely determined by ordinary static method. This can be generalized to three dimensions and multi-moored cables. The essence of this statement can be summarized as follows:

1. If the external forces applied to the arrays are constant, then the components of the resultant force ( $R_{m,n}^X, R_{m,n}^Y, R_{m,n}^Z$ ) acting in the  $(m,n)$  th cable segments can be given in terms of the applied external force through the following expressions.

for  $m = M(n)$  and  $n = 2, 3$

$$R_{M(n),n}^X = F_{M(n),n}^X \quad (27,1a)$$

$$R_{M(n),n}^Y = F_{M(n),n}^Y \quad (27,1b)$$

and

$$R_{M(n),n}^Z = F_{M(n),n}^Z \quad (27,1c)$$

for  $m = M(1)$  and  $n = 1$ , the branch point.

$$R_{M(1),1}^X = F_{M(1),1}^X + R_{2,2}^X + R_{2,3}^X \quad (28,1a)$$

$$R_{M(1),1}^Y = F_{M(1),1}^Y + R_{2,2}^Y + R_{2,3}^Y \quad (28,1b)$$

$$R_{M(1),1}^Z = F_{M(1),1}^Z + R_{2,2}^Z + R_{2,3}^Z \quad (28,1c)$$



and for  $m = 2, 3 \dots M(n)-1$  and  $n = 1, 2, 3$

$$R_{m,n}^x = F_{m,n}^x + R_{m+1,n}^x \quad (29,1a)$$

$$R_{m,n}^y = F_{m,n}^y + R_{m+1,n}^y \quad (29,1b)$$

$$R_{m,n}^z = F_{m,n}^z + R_{m+1,n}^z \quad (29,1c)$$

Thus starting from the secondary anchor points the forces are very readily summed up to give the reactions at every station.

2. Once the resultant force has been found, the tension  $T_{m,n}$  is given by:

$$T_{m,n} = \sqrt{(R_{m,n}^x)^2 + (R_{m,n}^y)^2 + (R_{m,n}^z)^2} \quad (30,1)$$

The tension in each segment results in the elongation of segments and the stressed length is obtained by

$$BL_{m,n} = BL_{o,m,n} \left( 1 + \frac{T_{m,n}}{XTEN(m,n)} \right) \quad (31,1)$$

where  $BL_{o,m,n}$  is unstressed length.

3. The next step is to find coordinates of each station. To start the solution, one starts from the primary anchor because its coordinates are known and are fixed. The coordinates, then, are found from the following equations:

$$X_{m,n} = \frac{BL_{m,n}}{T_{m,n}} R_{m,n}^x + X_{m-1,n} \quad (32,1a)$$

$$Y_{m,n} = \frac{BL_{m,n}}{T_{m,n}} R_{m,n}^y + Y_{m-1,n} \quad (32,1b)$$

$$Z_{m,n} = \frac{BL_{m,n}}{T_{m,n}} F_{m,n}^z + Z_{m-1,n} \quad (32,1c)$$

where  $m = 2, 3 \dots M(n)$

4. In general, for a guessed set of imaginary reactions, the  $M(n)$  ends of cable 2 and 3 will not be at the true anchor points. As a measure of the distance of these ends from the correct anchor points, the positive error function  $E$  is defined as

$$E = \sum_{n=2}^3 \{ [a_n - X_{M(n),n}]^2 + [b_n - Y_{M(n),n}]^2 + [c_n - Z_{M(n),n}]^2 \} \quad (33,1)$$

$E$  will uniquely vanish when the true anchor point has been reached.

5. Let the imaginary reactions applied to the ends of cables 2 and 3 be recalculated as

$$(F_{M(n),n}^x)' = F_{M(n),n}^x + \Delta F_{M(n),n}^x \quad (34,1a)$$

$$(F_{M(n),n}^y)' = F_{M(n),n}^y + \Delta F_{M(n),n}^y \quad (34,1b)$$

$$(F_{M(n),n}^z)' = F_{M(n),n}^z + \Delta F_{M(n),n}^z \quad (34,1c)$$

where primes denote the new imaginary reactions and the additive forces are defined by

$$\Delta F_{M(n),n}^x = \frac{\delta}{\sqrt{E}} [a_n^x M(n),n] \quad (35,1a)$$

$$\Delta F_{M(n),n}^y = \frac{\delta}{\sqrt{E}} [b_n^y M(n),n] \quad (35,1b)$$

$$\Delta F_{M(n),n}^z = \frac{\delta}{\sqrt{E}} [c_n^z M(n),n] \quad (35,1c)$$

The quantity  $\delta$  as defined earlier, is the positive convergence factor. Thus the correct equilibrium configuration is obtained once  $E \rightarrow 0$ .

Thus, in the above section the use of the Method of Imaginary Reactions to find equilibrium configuration of a tri-moored structure has been demonstrated. From here the method can be extended for n number of cables easily. (8)

#### D. APPLICATION OF METHOD OF IMAGINARY REACTIONS FOR POSITION-DEPENDENT EXTERNAL FORCES

In the preceding section the ability of the Method of Imaginary Reactions to determine the equilibrium configuration of a cable array, both single and multi-moored has been demonstrated. However, the use of this method to determine equilibrium configuration by simple statics was dependent totally upon the lumped external forces being constant.

However, underwater cable arrays are subjected to external forces due to weight and buoyancy, and hydrodynamic forces, position-dependent, like drag forces. They depend upon both the orientation and depths of the cable segments and on depths of elemental devices, with the result that the Method of Imaginary Reactions is not entirely applicable to the cable structure we wish to deal with in this report.



This problem can be solved by combining the Method of Imaginary Reactions with the Method of Successive Approximations.<sup>(3)</sup> Using this combined technique, the equilibrium configuration for arbitrary current profiles can be generated to any desired degree of accuracy. Essentially, this combined technique consists of making an initial guess as to the values of the hydrodynamic forces and uses these values to find the equilibrium position of the structure by the Method of Imaginary Reactions. Once this position is found, the hydrodynamic forces are recalculated, and the position of the array under the new forces is again found by the imaginary reactions. This iterative procedure is continued until the equilibrium configuration has been obtained to within a specified accuracy.

One of the ways to find a measure of accuracy for the successive approximation routine is to compare the equilibrium coordinates of any cable station for two successive iterations. If the coordinates differ by less than a fixed amount, the iteration will be considered satisfied and if any coordinate change is greater than this fixed amount, the iteration is continued. Analytically, if COMPD denotes this fixed accuracy value, then the successive approximation routine is considered satisfied when

$$|\theta_{m,n}^i - \theta_{m,n}^{i-1}| \leq \text{COMPD} \quad (36,1)$$

for the main array where  $\theta^i = x, y$  or  $z$  represents the equilibrium coordinates of any cable station obtained from the (i)th successive approximation routine. Thus, the combined technique offers the advantage that it can be used for the position dependent forces. In the following sections, this technique has been used to analyze the problem of a redundant structure.

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\*A brief discussion of elemental devices is made in Chapter III.

## CHAPTER II

### A NEAR-HORIZONTAL CABLE ELEMENT IN A MULTI-MOORED ARRAY

The major thrust of this investigation has been to study and analyze the tri-moored structure that has two of the cables connected by a horizontal cable. (This cable is called a tie leg in this report). A technique is developed in this report that enables the Method of Imaginary Reactions to be used for such a structure. More recently, Skop and O'Hara have extended the work reported herein to the general redundant structure.<sup>(12)</sup>

#### A. THE BASIC APPROACH

Let one first consider the two legs of a tri-moored buoy system, to which a tie leg has been attached. This structure, then, looks like an "A". The "A" structure can further be divided into two coupled parts: 1) the lambda ( $\Lambda$ ) structure and, 2) the tie leg structure. Neglecting the bending stiffness of the cables, there are only two conditions of coupling between the lambda structure and tie leg.

1. The geometrical compatibility condition
2. The force balance condition

All that is required is a simple lambda structure subjected to the above constraints between ( $\Lambda$ ) structure and tie leg, and then the structure will behave like an "A" structure.

#### B. THE GEOMETRICAL COMPATIBILITY CONDITION

Let one start with a lambda ( $\Lambda$ ) structure having A as an apex and B and C as anchor points. (See Figure 6) Points D and E represent approximately the position at which one wishes a tie leg inserted in the

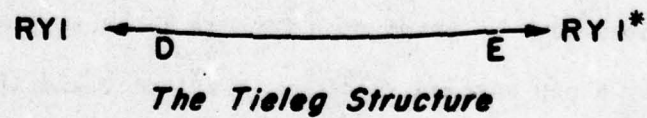
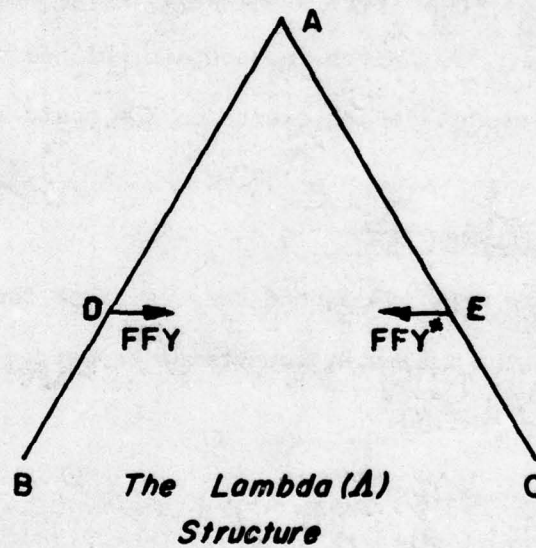
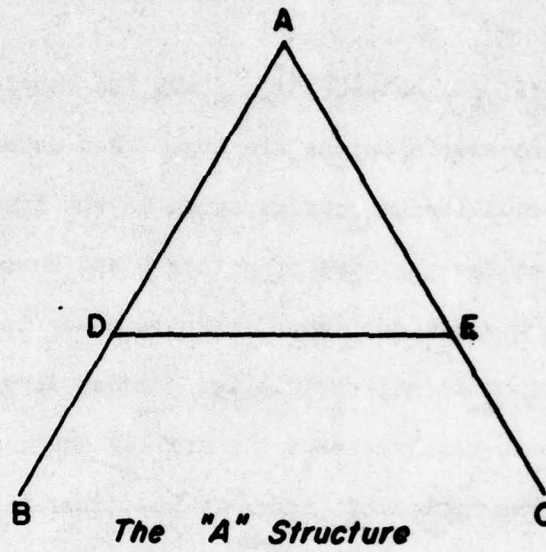


Figure 6: Representation of the Basic Tie Leg Concept



structure.  $(FFY)_{at D}$  and  $(FFY^*)_{at E}$  are the total resultant forces that the lambda structure exerts on the tie leg. Then using the Method of Imaginary Reactions the equilibrium configuration of the structure is obtained which, in result, gives the position of points D and E under loads.

If points D and E are known, then in order to meet the geometrical compatibility constraint, the tie leg chordal length must fit these points. It is, therefore, required that the tie leg shall have these points as end constraints. The Method of Imaginary Reactions is then applied to the tie leg by releasing the E end and the equilibrium configuration of the tie leg is obtained. This gives one certain end reactions  $(EY1)_{at D}$  and  $(RY1^*)_{at E}$  which the tie leg exerts on the basic lambda structure at points D and E.

#### C. THE FORCE BALANCE CONDITION

In order to meet the second constraint of force balance, it is important that the following conditions be met:

$$-(RY1)_{at D} = (FFY)_{at D} \quad (1,2a)$$

$$(RY1^*)_{at E} = -(FFY^*)_{at E} \quad (1,2b)$$

When this condition has been met, the complete solution to the problem has been obtained, i.e., the lambda structure is behaving as an "A" structure.

However, in case the above mentioned condition is not met, then the whole process must be repeated. That is  $(FFY)_{at D}$  and  $(FFY^*)_{at E}$  is replaced by a new assumed reaction  $(FFY)'_{at D}$  and  $(FFY^*)'_{at E}$  and new positions of points D and E are found under the action of these forces. Then, these positions are taken as reference points for finding the equilibrium position of the tie leg, which gives one the end reaction. This process is repeated until the force balance condition has been met. Once this

happens the two constraints have been taken into consideration and the problem is solved.

To reduce the number of iterations so as to satisfy the above constraint rather quickly, a convergence algorithm is developed in Chapter IV. This algorithm utilizes the values of FFY and RY1, and if force balance constraint is not satisfied, the routine employs a technique by which a new value of FFY' ( $FFY < FFY' < RY1$ ) and FFY\*' ( $FFY* < FFY*' < RY1*$ ) are obtained. This and subsequent values of FFY' and FFY\*' assure convergence to the correct values of forces, which satisfy the force balance condition, in minimum number of iterations.

## CHAPTER III

### MODELING AND ANALYSIS OF THE INTERNALLY REDUNDANT ARRAY SYSTEM

In this chapter, a brief description of the techniques used to model and analyze the array system is presented. The chapter has been divided into various sections, each of which deals with one phase of the modeling process. Further, each section deals separately with the main structure and the tie leg structure.

#### A. A FEW GUIDELINES

In this section a few guidelines for modeling the array are presented:

1. The tri-moored structure consists of a subsurface buoy, anchored to the bottom by three cables. These cables are referred to by number  $n$  where  $n = 1, 2$  and  $3$ . These cables are broken up into  $M(n)$  stations.
2. The primary anchor, the anchor for cable  $n = 1$ , is represented by  $(1,1)$ . All other stations are referred to by subscript pair  $(m,n)$  where  $m = 2, 3, \dots, M(n)$  and  $n = 1, 2$ , or  $3$  except for the secondary anchors which are referred to by  $(M(2),2)$  and  $(M(3),3)$ . The branch point at the subsurface buoy, in this case, is represented by  $(M(1),1)$ . This is the same point as  $(1,2)$  or  $(1,3)$  but by convention is represented by  $(M(1),1)$ .
3. The tie leg is attached to cables 2 and 3, at stations  $(L,2)$  and  $(L,3)$  respectively where  $L$  is any station between 1 and  $M(n)$ . The tie leg is further broken up into  $MN$  stations. Both arrays are represented diagrammatically as follows:



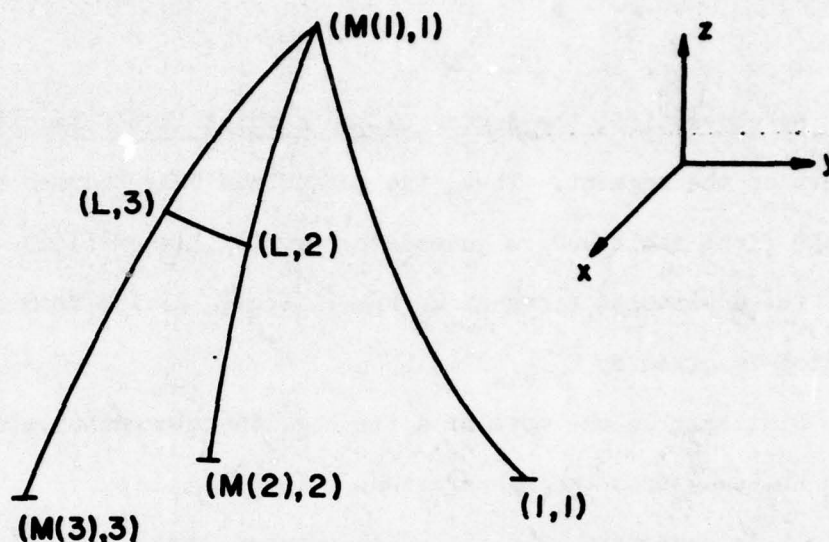


Figure 7: Nomenclature for the Tri-moored Array Structure with a Tie Leg

4. Since the external forces acting on the cables have to be lumped at these stations, each cable segment between the stations behaves like a straight line. The following guidelines are used for lumping these forces
  - i. Each point of discontinuity in a physical property of a cable is represented by a different station. Thus, each cable in the array has constant physical properties.
  - ii. As many additional stations are used as necessary to obtain a successful approximation to the continuous equilibrium shape of the array. However, the method of analysis does not depend upon the number stations and any ultrafine representation of the cable does not help.

#### B. DISCRETE ELEMENT NOMENCLATURE

The method of analysis has the ability to take into account discrete elements in the cable. These discrete elements exert a force on the system. The external forces, because of these elements, are lumped also at the stations neighboring them. Half segment lumping technique<sup>(5)</sup> is used here.

The objects which are attached to the  $(m,n)$ th cable segment on the main structure are indexed by  $(k,m,n)$  where  $k = 1, 2 \dots k(m,n)$ , counting in the direction as shown in Figure 8. For the tie leg--the objects are indexed by  $(j,m)$  where  $j = 1, 2 \dots j(m)$ .

By convention, the device on the station itself becomes the last object on the segment. Thus, the subsurface buoy becomes the last device on the first cable and is indexed by  $(k(M(1),1)), M(1),1)$ .

The unstressed distance of the  $(k,m,n)$ th device from the  $(m,n)$ th station is given by  $\bar{S}_{k,m,n}$

Similarly in the case of a tie leg, the element  $(j,m)$ th would be  $\bar{S}_{j,m}$  ft, away from the  $m$ th station.

It is necessary to distinguish between forces that are lumped at the lower segment from the forces that are lumped at the upper segment. Half segment technique<sup>(5)</sup> is used to lump these forces on the respective stations.

The objects that are attached to the half segment adjoining the  $(m,n)$ th station would then be differentiated from the objects adjoining  $(m+1,n)$ th segment as follows:

Let  $k(m,n)$  represent the value of  $k$  such that

$$\bar{S}_{k,m,n} < \bar{BL}_{om,n}/2$$

for  $k = 1, 2, \dots, \bar{k}(m,n)$

$$\bar{S}_{k,m,n} > \bar{BL}_{om,n}/2$$

for  $k = \bar{k}(m,n)+1, \dots, k(m,n)$

Similarly for a tie leg let  $j(m)$  represent the value of  $j$  such that

$$\bar{S}_{j,m} < L_m/2$$

and

for  $j = 1, 2, \dots, \bar{j}(m)$

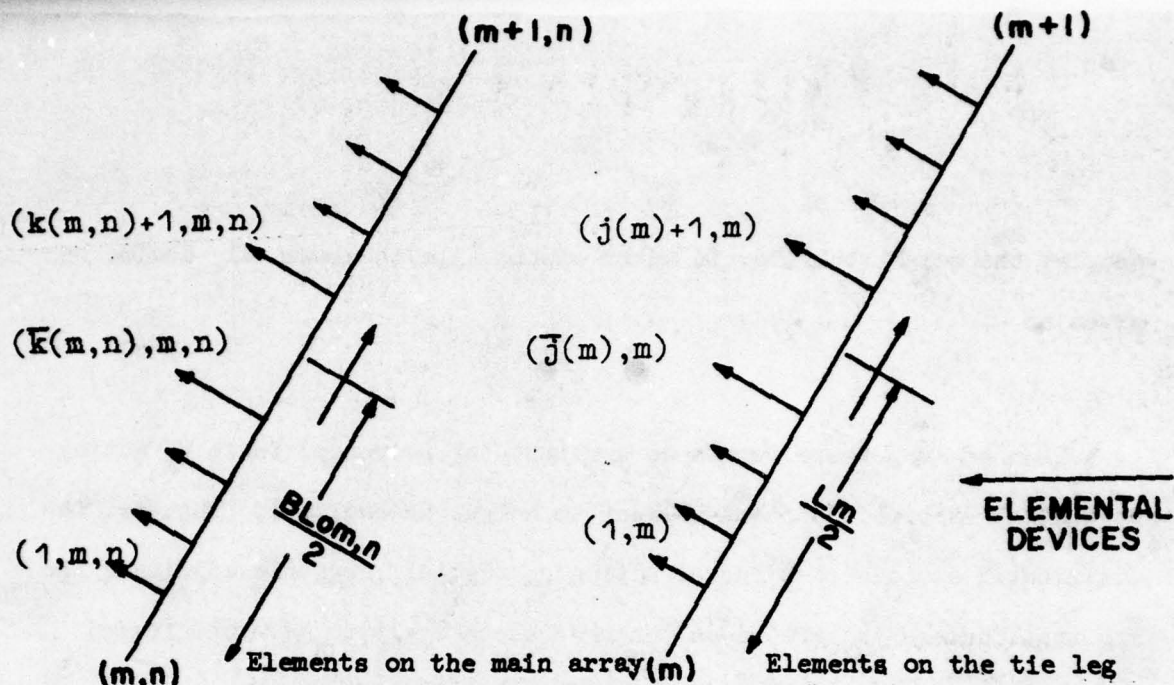


Figure 8: Representation of the Discrete Elemental Devices

These equations are diagrammatically represented by the figures shown above.

#### C. FORCES ACTING ON THE ARRAY SYSTEM

The total forces acting on the array system are made up of two factors:

1. The weight and buoyancy forces--here weight and buoyancy of discrete elements as well as of the cable are taken into account.
2. The hydrodynamic drag forces which are acting because of the interface between the cable and the current acting in water.

##### 1. Weight and Buoyancy Forces

In this section only the effect of weight and buoyancy is considered.

The weight and buoyancy actually can be differentiated only by the direction in which they act. The weights act downwards and buoyancy acts upwards. The Z direction is considered positive upwards.

To compute the weight and buoyancy forces let the weight (or buoyancy) per unit length in water of the  $m$ th cable segment be given by

$$W_m^c$$



and let the weight buoyancy in water of the  $(j,m)$ th elemental device be given as

$$w_{j,m}^e$$

Then to lump these forces as a weight (or buoyancy) force  $W_m$  acting at the  $m$ th station, the half segment technique is employed. That is, the distributed and discrete forces acting on the half segments adjoining the  $m$ th stations are integrated and summed respectively to give the lumped forces at that station.

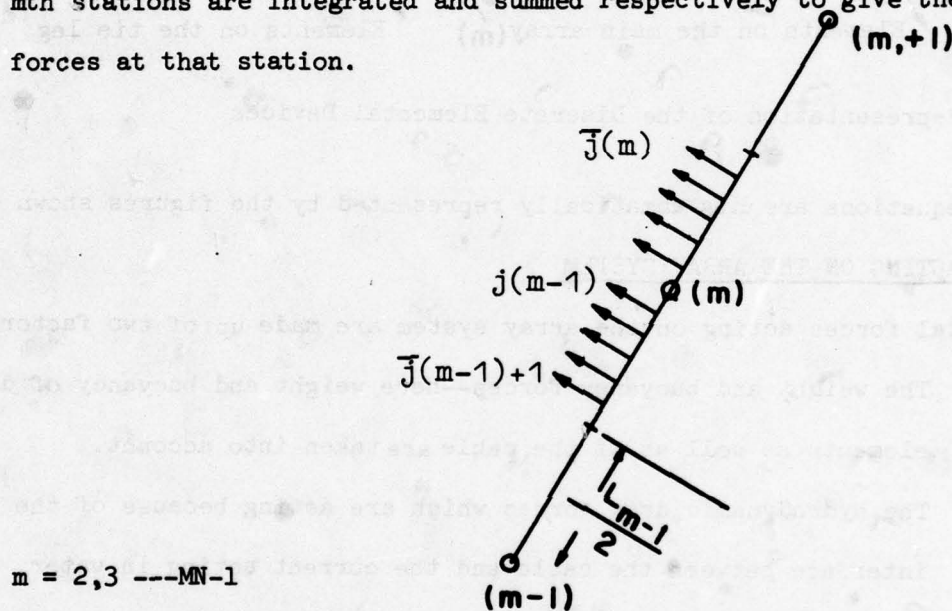


Figure 9: Representation of Lumped Force on a Station Using Half Segment Technique

Then because of the discrete element the weights that are lumped at station  $m$ , acting in the half segment of the  $(m-1)$  segment as shown in Figure 9 are expressed by

$$\sum_{j=j(m-1)+L}^{j(m-1)} w_{j,m-1}^e \quad (1,3)$$

Similarly the forces that are lumped at station  $m$  because of weights acting at the lower half segment of the  $m$ th segment are expressed by

$$\sum_{j=1}^{\bar{j}(m)} W_{j,m}^e \quad (2,3)$$

Also, the lumped force at station mth because of the weight (buoyancy) of the cable would similarly be

$$\frac{1}{2} W_{m-1}^c L_{m-1} + \frac{1}{2} W_m^c L_m \quad (3,3)$$

Such that total force lumped at mth station due to weights is given by

$$W_m = \sum_{j=\bar{j}(m-1)+1}^{\bar{j}(m-1)} W_{j,m-1}^e + \sum_{j=1}^{\bar{j}(m)} W_{j,m}^e + \frac{1}{2} [W_{m-1}^c L_{m-1} + W_m^c L_m] \quad (4,3)$$

Exactly the same discussion holds good for lumping the weights as forces in case of the tri-moored buoy structure. In this case, however, another factor n, the number of cables, has also to be taken into account.

Then for the tri-moored structure

$$W_{m,n} = \sum_{k=\bar{k}(m-1,n)+1}^{\bar{k}(m-1,n)} W_{k,m-1,n}^e + \sum_{k=1}^{\bar{k}(m,n)} W_{k,m,n}^e + \frac{1}{2} [W_{m-1,n}^c BL_{om-1,n} + W_{m,n}^c BL_{om,n}]$$

Where

m = 2,3,4---M(n)-1.

n = 1,2,3.

(5,3)

and for  $m = M(1)$   $n = 1$ , the subsurface buoy

$$k(M(1), 1) \quad \bar{k}(2,2)$$

$$W_{M(1),1} = \sum W_{\bar{k},M(1),1}^e + \sum W_{\bar{k},2,2}^e$$

$$k = \bar{k}(M(1),1)+1 \quad k=1$$

$$\bar{k}(2,3)$$

$$+ \sum W_{k,2,3}^e$$

$$k = 1$$

$$+ \frac{1}{2} [W_{M(1),1}^c \text{BL}_{OM(1),1} + W_{1,2}^c \text{BL}_{ol,2} + W_{1,3}^c \text{BL}_{ol,3}] \quad (6,3)$$

which are various expressions for representing the weights as lumped forces on the stations. In deriving this expression, however, the density of the water has been assumed to be constant.

## 2. Modeling of a Current Profile

Weight and buoyancy forces are one type of external forces. There are additional external forces which act on the cable because of the hydrodynamic interaction between the cable and an ocean current. This interaction produces drag forces which are dependent upon the velocity and direction of the local ocean current i.e., the angle at which local ocean current attacks the cable.

The method of analysis developed herein makes no restrictions on the shape of the current profile. For the sake of convenience, however, the following assumptions are made:

1. The ocean current, though depth dependent in magnitude, is uni-directional and normal to the direction of gravity.
2. The drag force component which acts in the direction normal to both the stream and the cable is zero.

The first assumption is made because most design currents are given as depth dependent and uni-directional. The second assumption is made because



of the limited experimental data available to support a general analytic expression for the side component of "drag" on cables.

The first assumption dictates that the current possess no Z component. If the angular direction with respect to the x axis is denoted by  $\phi$ , the expression for current in general can be written as

$$\vec{V} = V(Z) [ \vec{i} \cos \phi + \vec{j} \sin \phi ] \quad (7,3)$$

Where

$V(Z)$  = Magnitude of current profile at a height Z above the deepest anchor ( $Z=0$ ).

$\vec{i}$  = Unit vector in x direction

$\vec{j}$  = Unit vector in y direction

### 3. Hydrodynamic Coordinates

In order to calculate the various drag forces acting on the system, it is convenient to transform into a new set of natural hydrodynamic coordinates, defined with respect to direction of the cable and the current.

The following analysis is presented for the tri-moored structure. However, the same approach is used for the tie leg array.

1. In order to find a natural hydrodynamic coordinate system, we proceed in the following manner:

- i. Let  $\tau_{m,n}$  be the unit tangent to the (m,n)th cable segment, considered positive in the dir<sup>n</sup> of increasing m

- ii. Then

$$\pi_{m,n} = \frac{\vec{V} \times \tau_{m,n}}{|\pi_{m,n}|} \quad (8,3)$$

where  $\pi_{m,n}$  is a unit vector normal to both the (m,n)th cable segment and the current and

- iii.  $\eta_{m,n} = \tau_{m,n} \times \pi_{m,n}$  is a unit vector normal to the cable and lying in the plane that includes the (m,n)th cable segment and the stream.

In order to express the hydrodynamic base reference in terms of basic x, y, z reference frame, we obtain the following relation:

2. The unit vector  $\vec{\tau}_{m,n}$  is expressed in terms of

$$\vec{\tau}_{m,n} = \alpha_{m,n} \vec{i} + \beta_{m,n} \vec{j} + \gamma_{m,n} \vec{k} \quad (9,3)$$

where  $\alpha_{m,n}$ ,  $\beta_{m,n}$  and  $\gamma_{m,n}$  are the dir<sup>n</sup> cosines of the (m,n)th cable segment defined by

$$\alpha_{m,n} = \frac{X_{m+1,n} - X_{m,n}}{BL_{m,n}} \quad (10,3a)$$

$$\beta_{m,n} = \frac{Y_{m+1,n} - Y_{m,n}}{BL_{m,n}} \quad (10,3b)$$

$$\gamma_{m,n} = \frac{Z_{m+1,n} - Z_{m,n}}{BL_{m,n}} \quad (10,3c)$$

3. To express  $\vec{\pi}_{m,n}$  in terms of basic x,y,z coordinates we proceed as follows:

$$\vec{\pi}_{m,n} = \frac{V \chi \vec{\tau}_{m,n}}{|\vec{\pi}_{m,n}|}$$

and

$$V \chi \vec{\tau}_{m,n} = V(z) [\vec{i} \cos \phi + \vec{j} \sin \phi] \times [\alpha_{m,n} \vec{i} + \beta_{m,n} \vec{j} + \gamma_{m,n} \vec{k}]$$

$$= V(z) \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos \phi & \sin \phi & 0 \\ \alpha_{m,n} & \beta_{m,n} & \gamma_{m,n} \end{vmatrix}$$

$$= V(z) [\vec{i} (\gamma_{m,n} \sin \phi) - \vec{j} (\gamma_{m,n} \cos \phi) + \vec{k} (\beta_{m,n} \cos \phi - \alpha_{m,n} \sin \phi)]$$

Also  $\left| \vec{v} \times \tau_{m,n} \right| = \left| \vec{v} \right| \left| \tau_{m,n} \right| \Delta_{m,n}$

Where

$\Delta_{m,n}$  is the sine of the angle between the (m,n)th cable segment and the stream and is given by

$$\Delta_{m,n} = \sqrt{\gamma_{m,n}^2 + (\beta_{m,n} \cos \phi - \alpha_{m,n} \sin \phi)^2}$$

Also

$$\left| \vec{v} \right| = v(Z)$$

And

$$\left| \tau_{m,n} \right| = \sqrt{\alpha_{n,m}^2 + \beta_{m,n}^2 + \gamma_{m,n}^2} = 1$$

Thus

$$\begin{aligned} \pi_{m,n} = \frac{1}{\Delta_{m,n}} \{ & \vec{i}(\gamma_{m,n} \sin \phi) - \vec{j}(\gamma_{m,n} \cos \phi) \\ & + \vec{k}(\beta_{m,n} \cos \phi - \alpha_{m,n} \sin \phi) \} \end{aligned} \quad (11,3)$$

4. Similarly to express  $\eta_{m,n} = \pi_{m,n} \times \tau_{m,n}$  we proceed in the above manner to get

$$\begin{aligned} \eta_{m,n} = \frac{1}{\Delta_{m,n}} \{ & [(\gamma_{m,n}^2 + \beta_{m,n}^2) \cos \phi - \alpha_{m,n} \beta_{m,n} \sin \phi] \vec{i} \\ & + [(\gamma_{m,n}^2 + \alpha_{m,n}^2) \sin \phi - \alpha_{m,n} \beta_{m,n} \cos \phi] \vec{j} \\ & - [\gamma_{m,n} \beta_{m,n} \sin \phi + \gamma_{m,n} \alpha_{m,n} \cos \phi] \vec{k} \} \end{aligned}$$

#### 4. Hydrodynamic Coordinates for the Tie Leg

The tie leg is a single cable and as such, does not involve an index n, with the results the expressions (10,3), (11,3) and (12,3) are valid if



the index (n) is removed.

Thus the transformation expressions for hydrodynamic coordinates in case of a tie leg would be expressed as:

$$\tau_m = \alpha_m \vec{i} + \beta_m \vec{j} + \gamma_m \vec{k} \quad (13,3)$$

$$\pi_m = \frac{1}{\Delta_m} [\vec{i}(\gamma_m \sin \phi) - \vec{j}(\gamma_m \cos \phi) + \vec{k}(\beta_m \cos \phi - \alpha_m \sin \phi)] \quad (14,3)$$

$$\eta_m = \frac{1}{\Delta_m} [(\gamma_m^2 + \beta_m^2) \cos \phi - \alpha_m \beta_m \sin \phi] \vec{i} + (\gamma_m^2 + \alpha_m^2) \sin \phi - \alpha_m \beta_m \cos \phi \vec{j} - [\gamma_m \beta_m \sin \phi + \gamma_m \alpha_m \cos \phi] \vec{k} \quad (15,3)$$

## 5. The Hydrodynamic Distributed Forces

Once a set of hydrodynamic coordinates has been obtained, it becomes a simple matter to represent the hydrodynamic forces for unit length which are then resolved into  $\tau$ ,  $\pi$  and  $\eta$  directions.

### a) Side Force in the $\pi$ Direction

Pode has demonstrated some inherent difficulties that exist in defining hydrodynamic force.<sup>(6)</sup> Limited data is available to determine the functional forms of a side force in the  $\pi$  direction. However, it is known that magnitude of this force compared with the other two components is small and as such is neglected.<sup>(9)</sup>

Also  $\left| \vec{v} \times \tau_{m,n} \right| = \left| \vec{v} \right| \tau_{m,n} \Delta_{m,n}$

Where

$\Delta_{m,n}$  is the sine of the angle between the (m,n)th cable segment and the stream and is given by

$$\Delta_{m,n} = \sqrt{\gamma_{m,n}^2 + (\beta_{m,n} \cos \phi - \alpha_{m,n} \sin \phi)^2}$$

Also

$$\left| \vec{v} \right| = v(z)$$

And

$$\left| \tau_{m,n} \right| = \sqrt{\alpha_{n,m}^2 + \beta_{m,n}^2 + \gamma_{m,n}^2} = 1$$

Thus

$$\begin{aligned} \pi_{m,n} = \frac{1}{\Delta_{m,n}} \{ & \vec{i}(\gamma_{m,n} \sin \phi) - \vec{j}(\gamma_{m,n} \cos \phi) \\ & + \vec{k}(\beta_{m,n} \cos \phi - \alpha_{m,n} \sin \phi) \} \end{aligned} \quad (11,3)$$

4. Similarly to express  $\eta_{m,n} = \pi_{m,n} \times \tau_{m,n}$  we proceed in the above manner to get

$$\begin{aligned} \eta_{m,n} = \frac{1}{\Delta_{m,n}} \{ & [(\gamma_{m,n}^2 + \beta_{m,n}^2) \cos \phi - \alpha_{m,n} \beta_{m,n} \sin \phi] \vec{i} \\ & + [(\gamma_{m,n}^2 + \alpha_{m,n}^2) \sin \phi - \alpha_{m,n} \beta_{m,n} \cos \phi] \vec{j} \\ & - [\gamma_{m,n} \beta_{m,n} \sin \phi + \gamma_{m,n} \alpha_{m,n} \cos \phi] \vec{k} \} \end{aligned}$$

#### 4. Hydrodynamic Coordinates for the Tie Leg

The tie leg is a single cable and as such, does not involve an index n, with the results the expressions (10,3), (11,3) and (12,3) are valid if

b) Normal Drag Force in  $\eta$  Directions

It is known that the normal drag force for unit length which acts in the  $\eta$  direction has a magnitude given by

$$f_{m,n}^{\eta} = \frac{\rho}{2} C_{m,n}^N d_{m,n} (\vec{V} \cdot \eta_{m,n})^2 \quad (16,3a)$$

where

$\rho$  = the density of the fluid

$C_{m,n}^N$  = the coefficient of drag of (m,n)th cable segment

$d_{mn}$  = the diameter of (m,n)th cable segment

Expression for normal drag force in  $\eta$  direction for the tie leg array is essentially the same as (16,3a) except that no index n exists and as such is given by

$$f_m^{\eta} = \frac{\rho}{2} C_m^N d_m (\vec{V} \cdot \eta_m)^2 \quad (16,3b)$$

where  $C_m^N$  is the coefficient of drag of m th cable segment, the segment being normal to stream and  $d_m$  is the diameter of the mth cable segment.

c) Tangential Drag Force in  $\tau$  Direction

In most of the work done, this component is neglected being made equal to zero. However, as suggested by Skop and Kaplan,<sup>(9)</sup> this analysis assumes that this force can be given by the expression

$$f_{m,n}^{\tau} = \frac{\rho}{2} C_{m,n}^P d_{m,n} V(Z) [\vec{V} \cdot \tau_{m,n}] \quad (17,3a)$$

where

$C_{m,n}^P$  = the coefficient of drag of (m,n)th cable segment when the segment is parallel to the stream.

Similarly the tangential drag force for the tie leg is given by:

$$f_m^{\tau} = \frac{\rho}{2} C_m^P d_m V(Z) [\vec{V} \cdot \tau_m] \quad (17,3b)$$



where  $C_m^P$  is coefficient of drag of mth cable segment when the segment is parallel to the stream.

#### 6. The Total Hydrodynamic Force

From the above expressions, then, it becomes clear that the hydrodynamic force per unit length which is acting on the (m,n)th cable segment can be written as:

$$f_{m,n}^T = f_{m,n}^n + f_{m,n}^T \quad (18,3)$$

where

$$\begin{aligned} f_{m,n}^n &= \frac{\rho}{2} C_{m,n}^N d_{m,n} (V(Z) \Delta_{m,n})^2 \\ &= \mu_{m,n}^c \Delta_{m,n}^2 V^2(Z) \end{aligned} \quad (19,3)$$

and

$$\mu_{m,n}^c = \frac{\rho}{2} C_{m,n}^N d_{m,n} \quad (20,3)$$

also

$$\begin{aligned} f_{m,n}^T &= \frac{\rho}{2} C_{m,n}^P d_{m,n} V(Z) [\vec{V} \cdot \tau_{m,n}] \\ &= \mu_{m,n}^c r_{m,n}^D V^2(Z) [\alpha_{m,n} \cos \phi + \beta_{m,n} \sin \phi] \end{aligned} \quad (21,3)$$

where

$$r_{m,n}^D = \frac{C_{m,n}^P}{C_{m,n}^N} \quad (22,3)$$

Thus having known the expressions for  $f_{m,n}^n$  and  $f_{m,n}^T$  - these can be substituted into the equation (18,3) to get the total hydrodynamic force.

Similarly, the expression for total hydrodynamic force in the case of a tie leg

is given by

$$f_m^{\text{Total}} = f_m^{\eta} \eta_m + f_m^{\tau} \tau_m \quad (23,3)$$

where  $f_m^{\tau}$  and  $f_m^{\eta}$  are similar to Eqs. (21,3) and (19,3) with no (n).

To derive expressions for the projection of these hydrodynamic force in the i, j and k dir<sup>n</sup>, we proceed as follows:

The hydrodynamic force for the tri-moored structure acting in the  $\alpha$  dir<sup>n</sup> is given by

$$h_{m,n}^{c:x} = C_{m,n}^{c:x} v^2(z) \quad (23,3a)$$

Similarly

$$h_{m,n}^{c:y} = C_{m,n}^{c:y} v^2(z) \quad (23,3b)$$

$$h_{m,n}^{c:z} = C_{m,n}^{c:z} v^2(z) \quad (23,3c)$$

and for the tie leg by

$$h_m^{c:x} = C_m^{c:x} v^2(z) \quad (24,3a)$$

$$h_m^{c:y} = C_m^{c:y} v^2(z) \quad (24,3b)$$

$$h_m^{c:z} = C_m^{c:z} v^2(z) \quad (24,3c)$$

where  $C_{m,n}$ 's are known as the drag constants and they are defined by:

$$\begin{aligned} C_{m,n}^{c:x} = & \mu_{m,n}^c [\Delta_{m,n} (\gamma_{m,n}^2 + \beta_{m,n}^2) \cos \phi - \\ & \Delta_{m,n} \alpha_{m,n} \beta_{m,n} \sin \phi + \\ & r_{m,n}^D (\alpha_{m,n} \cos \phi + \beta_{m,n} \sin \phi) \alpha_{m,n}] \quad (25,3a) \end{aligned}$$

and

$$C_{m,n}^{c:y} = \mu_{m,n}^c [\Delta_{m,n} (\gamma_{m,n}^2 + \alpha_{m,n}^2) \sin \phi - \Delta_{m,n} \alpha_{m,n} \beta_{m,n} \cos \phi + r_{m,n}^D (\alpha_{m,n} \cos \phi + \beta_{m,n} \sin \phi) \beta_{m,n}] \quad (25,3b)$$

$$C_{m,n}^{c:z} = \mu_{m,n}^c [-\Delta_{m,n} (\alpha_{m,n} \cos \phi + \beta_{m,n} \sin \phi) \gamma_{m,n} + r_{m,n}^D (\alpha_{m,n} \cos \phi + \beta_{m,n} \sin \phi) \gamma_{m,n}] \quad (25,3c)$$

similarly for the tie leg

$$C_m^{c:x} = \mu_m^c [\Delta_m (\gamma_m^2 + \beta_m^2) \cos \phi - \Delta_m \alpha_m \beta_m \sin \phi + r_m^D (\alpha_m \cos \phi + \beta_m \sin \phi) \alpha_m] \quad (26,3a)$$

$$C_m^{c:y} = \mu_m^c [\Delta_m (\gamma_m^2 + \alpha_m^2) \sin \phi - \Delta_m \alpha_m \beta_m \cos \phi + r_m^D (\alpha_m \cos \phi + \beta_m \sin \phi) \beta_m] \quad (26,3b)$$

and

$$C_m^{c:z} = \mu_m^c [-\Delta_m (\alpha_m \cos \phi + \beta_m \sin \phi) \gamma_m + r_m^D (\alpha_m \cos \phi + \beta_m \sin \phi) \gamma_m] \quad (26,3c)$$

Once the hydrodynamic force on the cable array is found, the next stage is to lump these forces on the stations. To lump the distributed forces as a single force which acts at the (m,n)th cable station, the half segment lumping technique (5) is employed. Then using this technique, the lumped hydrodynamic forces are given by the equations:



for  $m = 2, 3 \dots M(n)-1$   $n = 1, 2, 3$

$$H_{m,n}^{c:\theta} = C_{m,n}^{c:\theta} \bar{V}(m,n: BL_{m,n/2}, BL_{m,n}) + C_{m+1,n}^{c:\theta} \bar{V}(m+1,n: 0, BL_{m+1,n/2}) \quad (27,3)$$

and for  $m = M(1)$  and  $n = 1$

$$\begin{aligned} H_{M(1),1}^{c:\theta} &= C_{M(1),1}^{c:\theta} \bar{V}(M(1),1: BL_{M(1),1/2}, BL_{M(1),1}) \\ &+ C_{1,2}^{c:\theta} \bar{V}(1,2: 0, BL_{1,2/2}) \\ &+ C_{1,3}^{c:\theta} \bar{V}(1,2: 0, BL_{1,3/2}) \end{aligned} \quad (28,3)$$

where  $\theta = x, y$  or  $z$

The function  $\bar{V}(m,n,\bar{\epsilon}_1,\bar{\epsilon}_2)$  represents the integral of  $(V^2(Z))$  along the  $(m,n)$ th half segment and is represented by

$$\bar{V}(m,n,\bar{\epsilon}_1,\bar{\epsilon}_2) = \int_{\bar{\epsilon}_1}^{\bar{\epsilon}_2} V^2[Z(m,n;\xi)] d\xi \quad (29,3)$$

where the argument  $z$  of  $V^2(Z)$  is expressed in terms of the integration parameter  $\xi$  along the  $(m,n)$ th cable segment through the relation

$$Z = Z(m,n;\xi) = Z_{m,n} + \gamma_{m+1,n} \quad (30,3)$$

where  $\gamma_{m+1,n}$  is the direction cosine of the  $m$ th segment as defined by equation (10,3c).

In the case of a tie leg the half segment technique yields the lumped hydrodynamic forces given by the following expression:

for  $m = 2, 3 \dots MN-1$

$$H_m^{c:\theta} = C_m^{c:\theta} \bar{V} (m: B_{m/2}, B_m) + C_{m+1}^{c:\theta} \bar{V} (m+1: 0, B_{m+1}) \quad (31,3)$$

where  $\theta = x, y$  or  $z$

and expression  $V (m, E_1, E_2)$  represents the integral of  $V^2(z)$  along the  $m$ th half segment and is represented by

$$\bar{V} (m, E_1, E_2) = \int_{E_1}^{E_2} V^2 [Z(m, \xi)] d\xi \quad (32,3)$$

#### D. DISCRETE ELEMENT HYDRODYNAMIC FORCES

The direction of drag on the discrete elements becomes parallel to  $V$  if it is assumed that there is no lift associated with them. The magnitude of the drag force in that case is given by:

$$\frac{\rho}{2} C_{k,m,n}^D A_{k,m,n} V^2(z) \quad (33,3)$$

where

$C_{k,m,n}^D$  = coefficient of drag of the  $(k,m,n)$ th elemental device

$A_{k,m,n}$  = effective cross-sectional area of  $(k,m,n)$ th elemental device

Then if this force be resolved in  $x, y$  and  $z$  coordinate system, the hydrodynamic force due to the  $(k,m,n)$ th elemental device is found as:

$$h_{k,m,n}^{e:x} = C_{k,m,n}^{e:x} V^2(z) \quad (34,3a)$$

$$h_{k,m,n}^{e:y} = C_{k,m,n}^{e:y} V^2(z) \quad (34,3b)$$

$$h_{k,m,n}^{e:z} = 0 \quad (34,3c)$$

where the drag constants of the  $(k,m,n)$ th elemental device are given by

$$C_{k,m,n}^{e:x} = \mu_{k,m,n}^e \cos \phi \quad (35,3a)$$

$$C_{k,m,n}^{e:y} = \mu_{k,m,n}^e \sin \phi \quad (35,3b)$$

and  $\mu_{k,m,n}^e$  being the hydrodynamic coordinate is defined by

$$\mu_{k,m,n}^e = \frac{\rho}{2} C_{k,m,n}^D A_{k,m,n} \quad (36,3)$$

Similarly, for the tie leg the hydrodynamic force due to the (j,m)th elemental device is given as

$$h_{j,m}^{e:x} = C_{j,m}^{e:x} V^2(Z) \quad (37,3a)$$

$$h_{j,m}^{e:y} = C_{j,m}^{e:y} V^2(Z) \quad (37,3b)$$

$$h_{j,m}^{e:z} = 0 \quad (37,3c)$$

where drag constants of the (j,m)th elemental device are given by

$$C_{j,m}^{e:x} = \mu_{j,m}^e \cos \phi \quad (38,3a)$$

$$C_{j,m}^{e:y} = \mu_{j,m}^e \sin \phi \quad (38,3b)$$

and  $\mu_{j,m}^e$  is the hydrodynamic coordinate defined by

$$\mu_{j,m}^e = \frac{\rho}{2} C_{j,m}^D A_{m,n} \quad (39,3)$$

The next step is to apply the half segment lumping technique to find the hydrodynamic forces acting at the (m,n)th station and due to discrete elemental devices. This is given through the following relations:

for  $m = 2, 3 \dots M(n) - 1$  and  $n = 1, 2, 3$

$$H_{m,n}^{e:\theta} = \sum_{k=\bar{k}(m,n)+1}^{k(m,n)} C_{k,m,n}^{E:\theta} V^2 [Z(m,n; S_{k,m,n})] +$$

$$\sum_{k=1}^{\bar{k}(m+1,n)} C_{k,m+1,n}^{e:\theta} V^2 [Z(m+1,n; S_{k,m+1,n})] \quad (40,3)$$

where  $\theta = x, y$  or  $z$

and for  $m = M(1)$  and  $n = 1$



$$H_{M(1),1}^{e:\theta} = \sum_{k=\overline{k(M(1),1)+1}}^{\overline{k(M(1),1)}} C_{k,M(1),1}^{e:\theta} V^2 [Z(M(1),1:S_{k,m(1),1})]$$

$$+ \sum_{k=1}^{\overline{k(1,2)}} C_{k,1,2}^{e:\theta} V^2 [Z(1,2:S_{k,1,2})]$$

$$+ \sum_{k=1}^{\overline{k(1,3)}} C_{k,1,3}^{e:\theta} V^2 [Z(1,3:S_{k,1,3})] \quad (41,3)$$

Again,  $\theta = x, y$  or  $z$  and argument  $z$  of  $(V^2(Z))$  is expressed in terms of the position of the  $(k,m,n)$ , the device along the  $(m,n)$ th segment through the relation

$$Z(m,n:S_{k,m,n}) = Z_{m-1,n} + \gamma_{m,n} S_{k,m,n} \quad (42,3)$$

Using half segment technique to find hydrodynamic forces acting at the  $m$ th station of the tie leg due to discrete elements, the following expression is derived:

$$H_m^{e:\theta} = \sum_{j=\overline{j(m)+1}}^{\overline{j(m)}} C_{j,m}^{e:\theta} [V^2 Z(m,S_{j,m})] \\ + \sum_{j=1}^{\overline{j(m+1)}} C_{j,m+1}^{e:\theta} V^2 [Z(m+1, S_{j,m+1})] \quad (43,3)$$

for  $m = 2, 3, \dots, MN-1$

where  $\theta = x, y$  or  $z$

#### E. FINAL EXTERNAL FORCES

Thus, to summarize the external force

$$(F_{m,n}^x, F_{m,n}^y, F_{m,n}^z)$$

that is acting at the  $(m,n)$ th cable station for the tri-moored array can be given

as:

$$F_{m,n}^x = H_{m,n}^{c;x} + H_{m,n}^{e;x} \quad (44,3a)$$

$$F_{m,n}^y = H_{m,n}^{c;y} + H_{m,n}^{e;y} \quad (44,3b)$$

and

$$F_{m,n}^z = W_{m,n} + H_{m,n}^{e;z} \quad (44,3c)$$

Where the lumped weight forces  $W_{m,n}$  are defined by Eqs. (5,3), the lumped hydrodynamic forces  $H_{m,n}^{c;\theta}$  due to drag forces on the cable segments are defined by Eqs. (27,3), and the lumped hydrodynamic forces  $H_{m,n}^{e;\theta}$  due to the drag forces on the elemental devices are defined by Eqs. (40,3). The above equations, however, do not apply to stations (M(2),2) and (M(3),3) as imaginary reactions act on these.

Similarly, the external force

$$(F_m^x, F_m^y, F_m^z)$$

acting at the  $m$  th cable station for the tie leg array can be given by the following equations:

$$F_m^x = H_m^{c;x} + H_m^{e;x} \quad (45,3a)$$

$$F_m^y = H_m^{c;y} + H_m^{e;y} \quad (45,3b)$$

and

$$F_m^z = W_m + H_m^{e;z} \quad (45,3c)$$

Where  $W_m$  is defined by Eqs. (4,3),  $H_m^{c;\theta}$  is defined by Eqs. (31,3) and  $H_m^{e;\theta}$  is defined by Eqs. (43,3).

In the above equations, only the weight forces are constant. The hydrodynamic forces depend on the position of the structure through both the orientation and the depth. As discussed in Chapter I, this problem is solved by using the Method of Imaginary Reactions in conjunction with the Method of Successive Approximations.

## CHAPTER IV

### THE CONVERGENCE ALGORITHM USING A BINARY SEARCH ROUTINE

In Chapter II it was suggested that a convergence algorithm is required to satisfy the force balance condition using a near-minimum number of subroutine iterations. The force balance constraints suggest that in order to obtain the tie leg effect in a lambda structure (refer to Fig. 6) the following equations must be satisfied.

$$FFY_{\text{at D}} = -RY1_{\text{at D}} \quad (1,4a)$$

$$-FFY^*_{\text{at E}} = RY1^*_{\text{at E}} \quad (1,4b)$$

where FFY and FFY\* are the resultant forces exerted by the main cable arrays on the tie leg and RY1 and RY1\* are the end reactions that the tie leg exerts on the main cable array.

If the above set of forces do not satisfy the force balance constraints, then a different value for FFY and FFY\* is used which produces a different chordal distance D'E' such that the tie leg fits into this new distance. Now, the force balance test is applied and if it is not satisfied, the above process is repeated.

This chapter discusses a technique which determines a new value for forces FFY and FFY\* so that the number of iterations, before forces converge to satisfy equations (1,4), is hopefully made minimal. The following discussion is restricted to force FFY and reaction RY1; however, the same discussion holds for force FFY\* and reaction RY1\*.

Various algorithms to find a new value of FFY (refer to Figure 6) were evaluated; however, a Binary Search Routine was found to be the most direct method to achieve fact convergence.



### A. BINARY SEARCH ROUTINE

A Binary Search Routine is an algorithm used to find a new value of FFY such that it always lies in between the previous two values of FFY and RY1. If the force FFY produces a reaction RY1 in the tie leg, then the new value of FFY is given by:

$$FFY' = \text{ABS} \left( \frac{\text{ABS}(RY1) - \text{ABS}(FFY)}{2} \right) + \text{Minimum}(\text{ABS}(FFY), \text{ABS}(RY1)) \quad (2,4)$$

This new value of FFY as given by FFY' and used with the proper sign on the tri-moored array, then, produces another value of RY1. The force balance criterion is applied and the process repeated until the condition is satisfied. A typical plot of the result of using this algorithm in a practical problem is as shown in Figure 10, where the convergence can be seen.

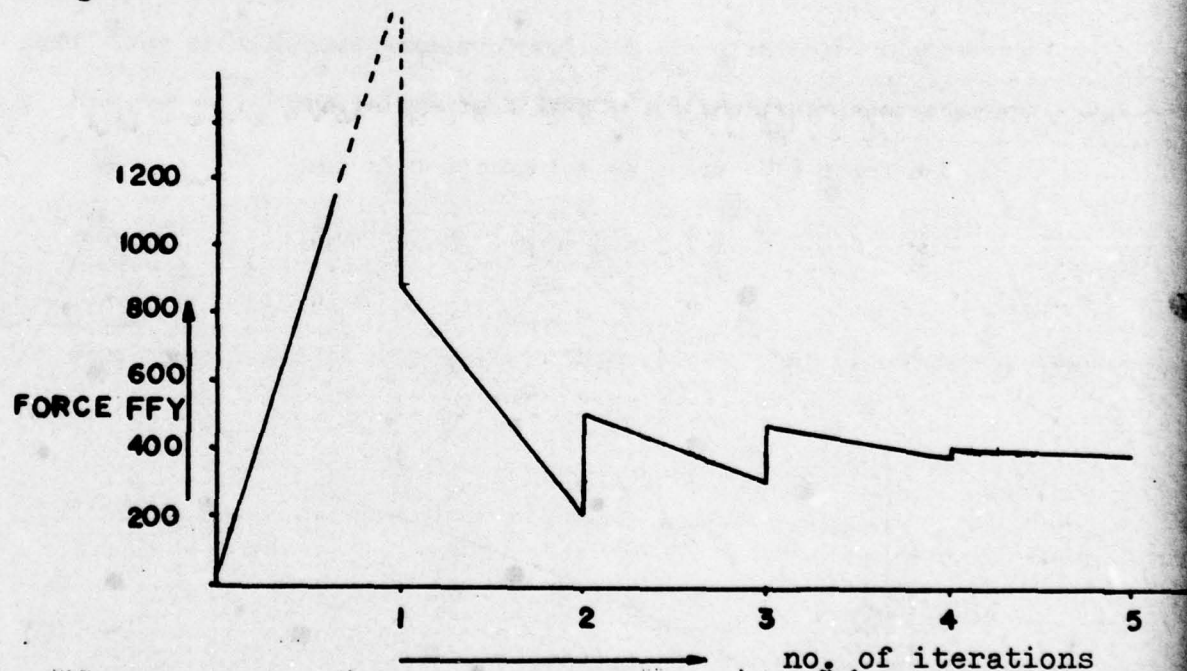


Figure 10: The Behavior of the Force FFY When Binary Search Routine  
Is Used

The details of the Binary Search Routine as used in the problem are listed below.

If  $FFY$  is the first assumed force that is applied on the main cable, then after equilibrium has been attained, a chordal length into which the tie leg is to fit is obtained. If this equilibrium configuration of the main structure is not compatible for the tie leg, then another assumed value  $FFY'$  ( $FFY' > FFY$ ) is used on the main cable which reduces the chordal distance by an amount  $X$  (see Figure 11).

Thus by several iterations a tie leg is made to fit into an appropriately adjusted chordal distance, as shown in Figure 12. This produces a reaction  $RY1$  in the tie leg. Now the force balance test is applied and if it is not satisfied, a new value of  $FFY''$  ( $FFY' < FFY'' < RY1$ ) is obtained from the Binary Search Routine using equation (2,4). This process is repeated until the force balance condition is met. When this happens equations (1,4) have been satisfied.

The force  $FFY$ , drawn as a function of  $x$  is:

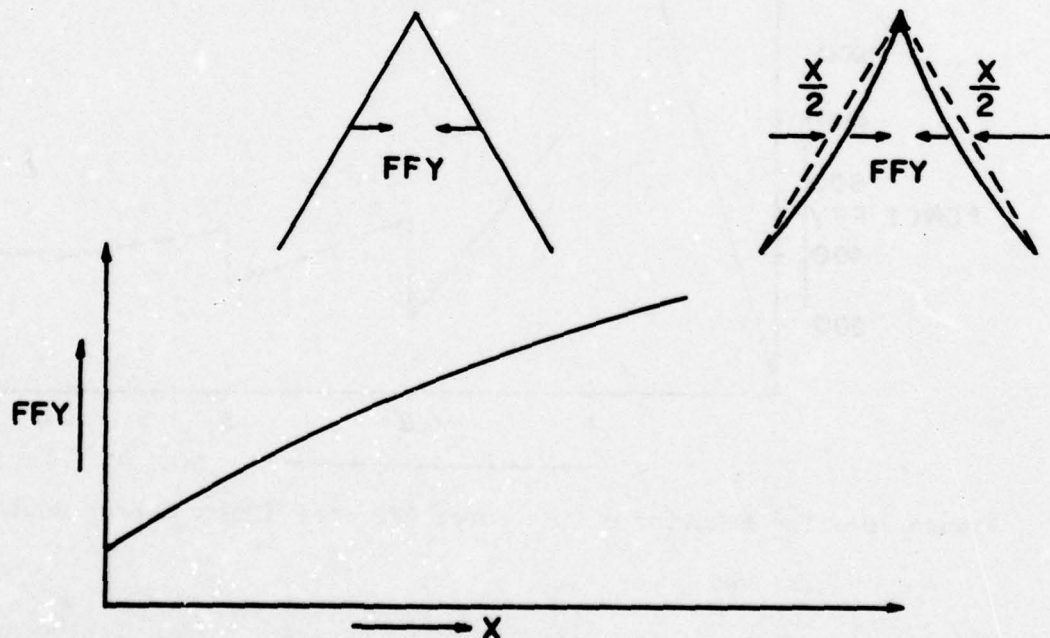


Figure 11: The Relationship Between the Force  $FFY$  and a Displacement  $X$ .

Similarly if a curve of reaction  $RYI$  is drawn with respect to  $X$ , it will behave as shown below:

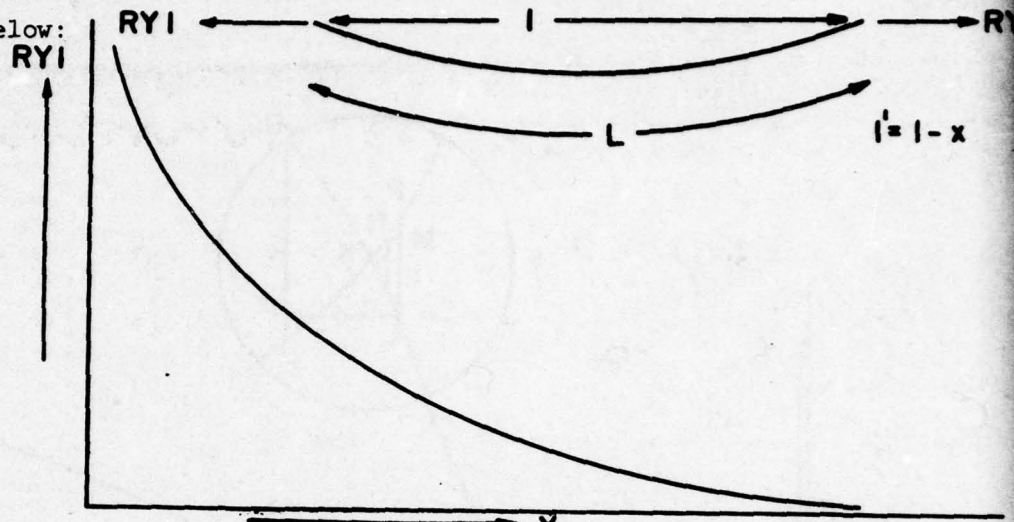


Figure 12: The Relationship Between the Reaction  $RYI$  and a Displacement  $X$ .

If both these curves are superimposed, a graph as shown in Figure 13 is obtained.

Then to summarize,  $FFY'$  produces the reaction  $RYI$  and this is represented by  $AB$  (notation follows Figure 13). Using binary search a new value of  $FFY$  is found and is represented by  $CD$ . Force  $FFY$  at  $D$  produces a reaction  $RYI$  given by  $E$  so that this case is represented by  $DE$ . Binary Search Routine is used again to find the new value of  $FFY$  represented by  $FL$ .

Force represented by  $FL$  is used to find the chordal distance in which the tie leg has to fit. A reaction represented by point  $P$  is required to do so. The Binary Search Routine is again used to find a new value for force  $FFY$ , which is now represented by  $MN$ . Thus, this process is repeated until the forces represented by point  $K$  are obtained. At this stage equations (1,4) are satisfied. The convergence to the required forces that satisfy the force balance condition, in all computer runs and tests made, appears to be very quick.

A block diagram representing the stepwise use of the Binary Search Routine is shown in Figure 14.





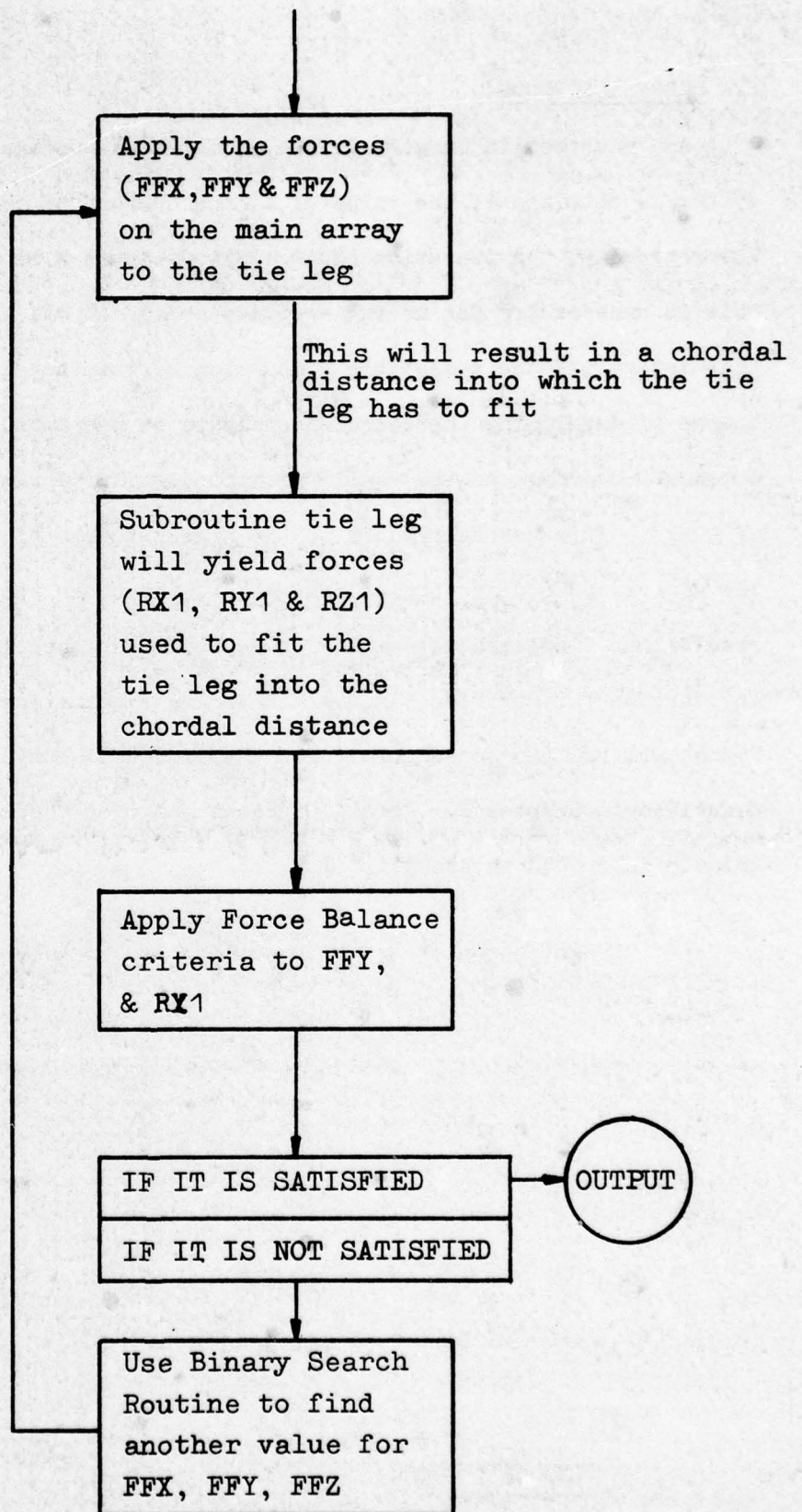


Figure 14: Block Diagram of the Binary Search Routine

## B. PRECISION FOCUS

As described in Chapter I, the equilibrium configuration of the array system is obtained if the value of  $E$ , the measure of error, is nearly zero. Theoretically, the iteration could continue until  $E$  is exactly zero. However, this is unnecessary for useful accuracy and a cut off value  $COMPE$  is defined that determines the acceptable completion of the iterative process.\* In the course of developing the computer analysis it was found that the tie leg array would not converge to its equilibrium configuration within specified limits of  $COMPE$ . This was attributed to the higher levels of required forces in the  $y$ -coordinate direction as compared to the other two directions, with the result that  $\delta$  becomes too small for rapid convergence in all three directions. To overcome this problem, another convergence technique, called Precision Focus, was used in conjunction with the method of convergence that has been described in Chapter I. Precision Focus has been used by Savage<sup>(13)</sup> and is briefly described in Appendix II.

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\* A brief discussion of  $COMPE$  appears in Appendix II.



## CHAPTER V

### TESTING THE COMPUTER MODEL

The computer program written to simulate the given array system is listed in Appendix I. This chapter presents a discussion of some testing of this computer model, and some aspects of the general behavior of the array system.

The computer model tested has the following specifications:

The tri-moored structure consisted of three identical legs, each 25,000 ft. long, in the unstressed state. These legs have a diameter of 0.675 inches and a weight in water of 0.606 lb/ft. The extensional rigidity of each cable segment is given as  $2.2 \times 10^6$  lb. The co-efficient of normal drag is assumed as 1.40 for the entire range of current velocities.

The tie leg cable has a diameter of 0.675 inches and an extensional rigidity of  $2.2 \times 10^6$  lb. The length and weight of the cable was determined internally by the program.

As described in Chapter II, the basic approach in the analysis of the given system has been to divide the array system into the following coupled parts:

1. The tri-moored array structure
2. The tie leg array structure

This approach was thought to be most practical because results obtained, namely the displacements of the subsurface buoy in the given system, could be compared readily with the displacements obtained from the analysis of a tri-moored structure without a tie leg for which a computer program has been published.<sup>(9)</sup> Also, it is possible to study the behavior of the main structure and the tie leg separately. To check the program output the following extreme cases

were analyzed on the computer.

1. When the tie leg is attached near the top of the tri-moored structure, where the tie leg has negligible length
2. When the tie leg is attached near the bottom of the tri-moored structure, where the tie leg is essentially connected between two of three anchor points

Since both of these are limiting conditions where the tensions will be taken up by the anchors in one case and where the tie leg is of negligible length in the other, the system should behave similarly to a simple tri-moored structure as shown in Figure 1. To look at these extreme conditions and at the general case of the tie leg attached anywhere on the main structure, the computer model was used and the results are discussed in the next three sections.

A. TESTING THE COMPUTER MODEL WHEN THE TIE LEG IS NEAR THE TOP OF THE TRI-MOORED STRUCTURE

The computer program is written in such a manner that it is not possible to place the tie leg precisely at the apex. To cope with this problem, it was decided to break the main cable into unequal segments such that the first and last segments were only of 5 feet length. Each of the cables was broken up into 20 segments.

This resulted in a simulation which allowed the tie leg to be put at a distance of five feet from the top. The displacements obtained from this configuration were used for comparison purposes. Many different conditions of tie leg parameters were tried and in each case, the tie leg structural analysis gave similar results to the already tested program for a simple tri-moored structure<sup>(9)</sup>. For example, with a tie leg of 6.12 feet in length and the weight per foot of cable at 0.505 lb./ft., the displacement of the buoy due to a standard test current, for the case of a tri-moored array without this tie leg is:

Horizontal deflection of the buoy = 36.95 ft.

Vertical deflection of the buoy = -11.50 ft.

The displacement of the buoy due to the same test current with the tie leg cable attached between two cables near the apex of the tri-moored system is:

Horizontal deflection of the buoy = 35.12 ft.

Vertical deflection of the buoy = 11.50 ft.

The results obtained from this test condition compare favorably with the results obtained from the analysis of a simple tri-moored structure.

**B. TESTING THE COMPUTER MODEL WHEN THE TIE LEG IS LOCATED NEAR THE BOTTOM OF THE TRI-MOORED STRUCTURE**

For this condition, the tie leg was attached near the bottom (five feet up each leg), between two of three anchor points.

The length of the tie leg was 30612.8 feet and weight of the cable per foot was 0.367 lb. This weight of the cable was determined internally by the program so as to make the tie leg array slightly positively buoyant.

The displacements of the buoy when placed in the test current with the tie leg are as follows:

1. Horizontal Deflection of the buoy = 39.61 ft.
2. Vertical Deflection of the buoy = -12.45 ft.

These results should be compared with the displacements of the standard tri-moored buoy, as listed above.

**C. PRELIMINARY STUDY USING THE COMPUTER MODEL**

Besides the above two test cases, the installed position of a tie leg was varied along the length of the main cables. This was done in order to study the behavior of the model for different positions of a tie leg. While the purpose of this report is not to conduct a complete study of



system behavior, enough data was gathered to predict the general behavior of a tie leg in a tri-moored structure. This behavior is presented in Figure 15 and 16 wherein the vertical horizontal deflections of the buoy are plotted against the position of the tie leg. The curves should be considered only as approximate since the data points were obtained from calculations using slightly different tie leg cable weights for each location. This was done as a matter of convenience. The error introduced does not alter the general behavior or the conclusion that horizontal deflections of the apex are increased by approximately a factor of two and the vertical deflections of the apex are increased by approximately a factor of four. If a prototype system is contemplated, a more detailed analysis should be conducted.

From the graphs it can be inferred that worse deflections, both horizontal and vertical, are encountered, as one would expect, when the tie leg is at a position near the middle. The current used is perpendicular to the tie leg.

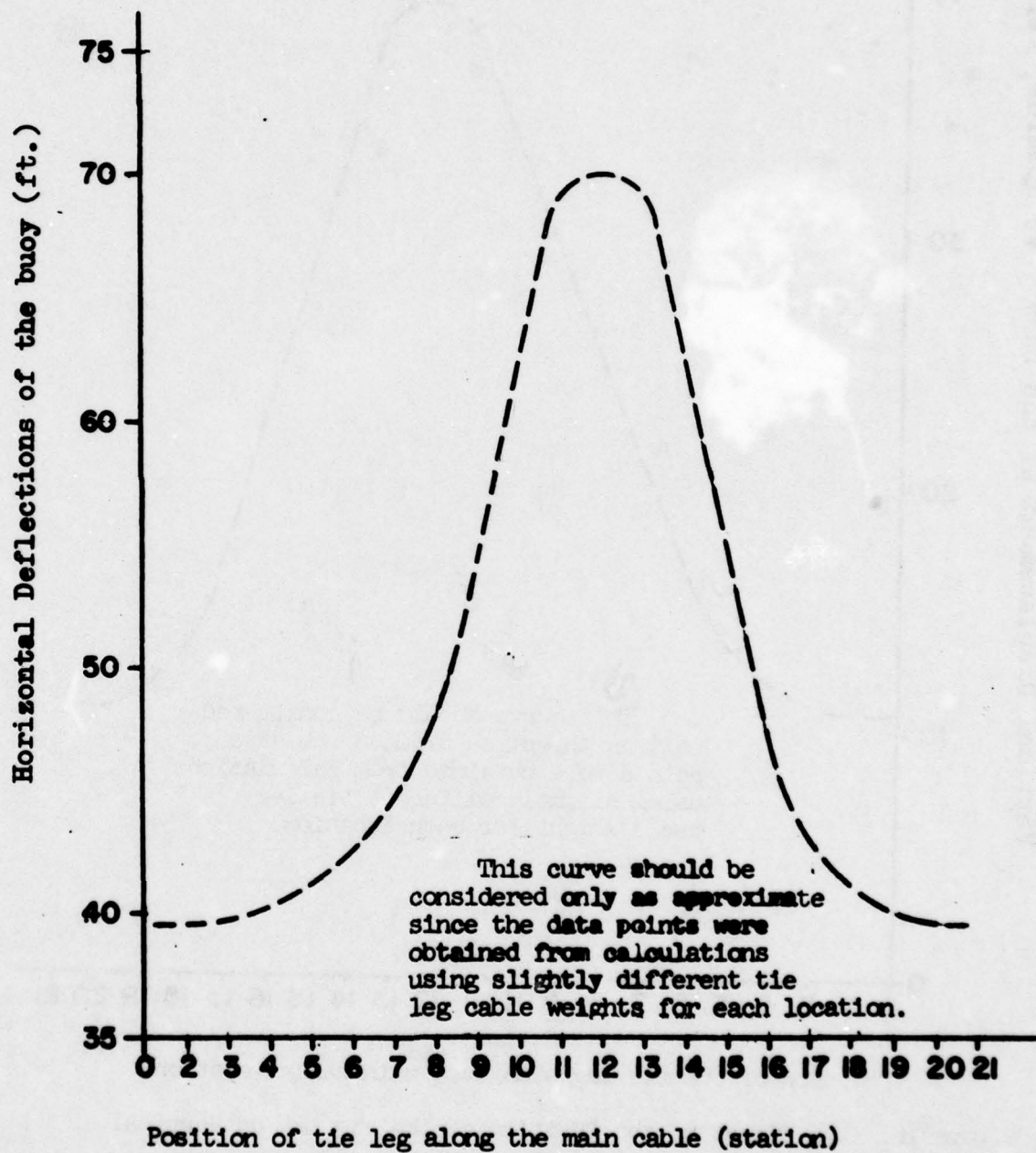


Figure 15: The Effect of the Position of the Tie Leg on Horizontal Deflection of the Buoy

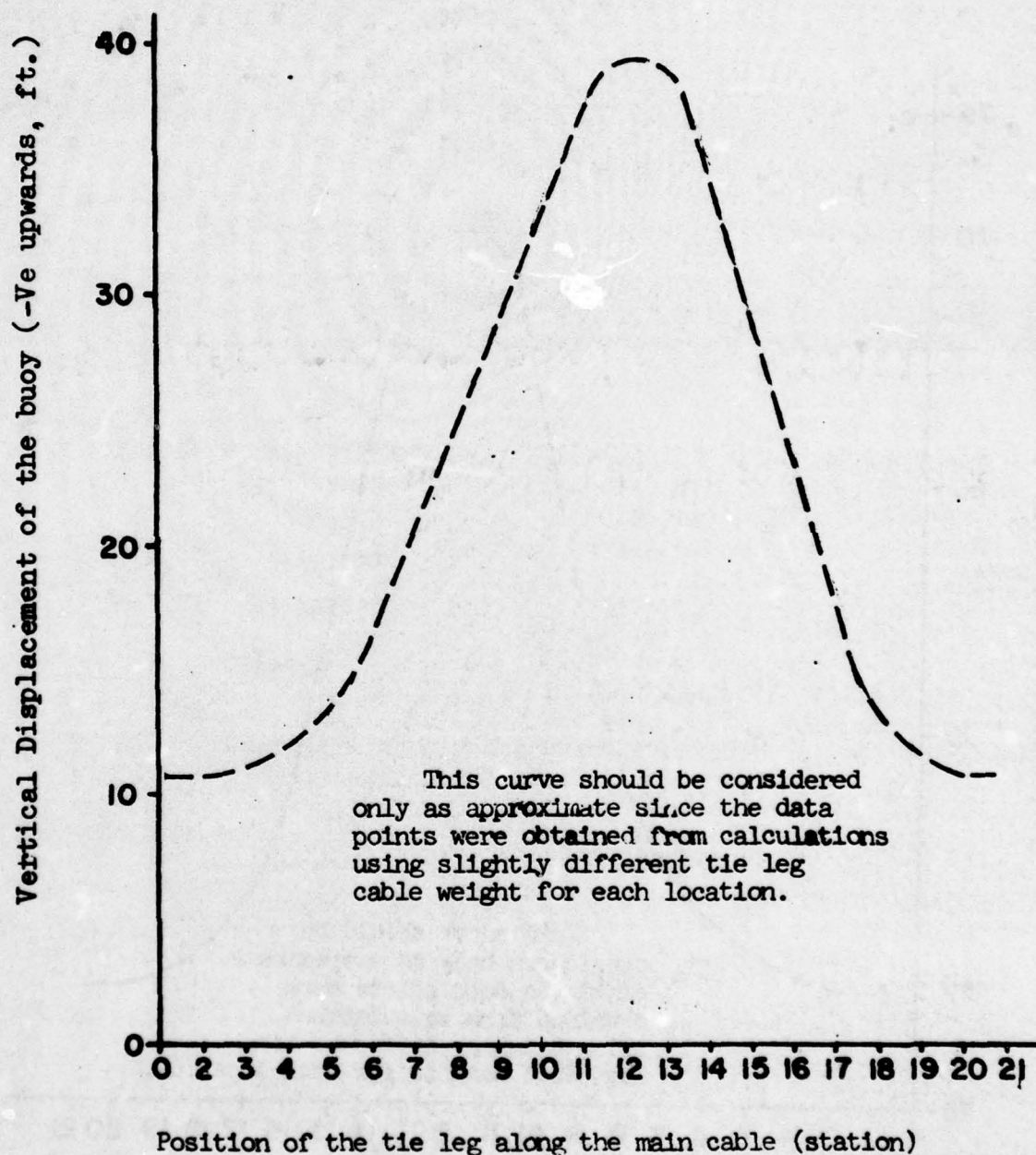


Figure 16: The Effect of the Position of the Tie Leg on Vertical Deflections of the Array



## APPENDIX I

### THE COMPUTER PROGRAM

#### A. DESCRIPTION

The computer program, required to simulate a tri-moored buoy with a tie leg is reproduced in this appendix. This program is written in FORTRAN IV and can be compiled and executed on most of IBM-360 facilities. The facility at the University of New Hampshire is IBM 360-40 and the computer program has been specifically adapted to it.

The program consists of 14 subroutines besides the main section. Comment cards at the beginning of each of these sections describe the nomenclature and in some cases the purpose of each subroutine.

The program, as written, is restricted to 20 segments and to 10 elemental devices per segment in each of the main arrays and to 21 segments and 5 elemental devices in each tie leg array. This is only for convenience and these numbers can easily be changed by changing the dimensions of the common arrays in the main program and in the subroutine and functions.

Since the length of the tie leg is dependent upon the pretensioning, the length of each segment is determined internally. The tie leg is made positively buoyant by about 2% and as such, the weight per unit feet is also computed internally.

The total length of the computer program is (FB58)<sub>16</sub>. If at any facility the computer memory is inadequate, then K and M dimensions for all the arrays are reduced to the largest values of KMAS (m,n) and JMAX(m) in the case of the tie leg--depending upon any particular analysis.

#### B. INPUT DATA CARDS

The complete input data is controlled by subroutine INPUT. Thus, data cards in the program correspond to this subroutine.

This subroutine has been broken up into four parts:

First part consists of reading in of data that is valid for the main arrays and the tie leg arrays.

Second part deals with data relevant only to the main arrays.

Third part consists of reading in of data for the tie leg array.

Finally, the last part deals with the profiles of the hydrodynamic current as it attacks the main array and the tie leg respectively.

1. The first input card contains: COMPE, COMPD, STAPSI, DELPSI, ENDPsi, and TIECOM. F10.3 FORMAT

COMPE	COMPD	STAPSI	DELPsi	ENDPSI	TIE COM
Comparison value for E- <u>Error</u> Function	Comparison value for displacement	First current angle to be analyzed in leg	Change of current angle in degrees	Final current angle in degrees	Comparison value to meet force balance criteria

2. The next three cards contain the Anchor Positions, AAl = (=X), BB1 (=Y), CC1 (=Z) -F 10.3 FORMAT

AAl(=X)	BB1(=Y)	CC1(Z)	
X of Anchor 1	Y of Anchor 1	Z of Anchor 1	card 2
X of Anchor 2	Y of Anchor 2	Z of Anchor 2	card 3
X of Anchor 3	Z of Anchor 3	Z of Anchor 3	card 4

3. The fifth card contains the number of stations per cable, MMAX(N) - I 5 FORMAT

MMAX(1)	MMAX(2)	MMAX(3)
No. of stations on cable 1	No. of stations on cable 2	No. of stations on cable 3

4. The next group of cards contain the physical properties of the cable segments and the discrete elements. This starts with cable 1, which goes from anchor point to subsurface buoy--followed by cables 2 and 3.

FORMAT OF CARDS DESCRIBING THE PROPERTIES OF THE CABLE SEGMENTS

BLBAR	WC(M,N)	XTEN(M,N)	TDRAG(M,N)
(M,N) F 10.2	F 10.2	F 10.2	F 10.2
Unstressed length (ft.)	Weight/ ft. (lb./ft.)	Extensional rigidity (lb.)	Normal drag coefficient

CABDIA (M,N) F 10.2	PDRAG(M,N) F 10.2	KMAX(M,N) I-5
Cable Dia. (Inches)	Parallel Drag Coeff.	No. of Discrete Elements

following each one of these is KMAX(m,n) cards giving the physical properties of the discrete elements

SBAR(k,m,n)	WE (k,m,n)	DRAG CF(k,m,n)	XREA(k,m,n)
Length segment (m-1,n) element	weight lb.	Drag. coeff.	X-sectional area

5. The next group of data cards contain input data for the tie leg array.

1. First card in this group contains the number of station on the main array to which tie leg is attached, - I 3 FORMAT

L - I 3
No. of stations on the main array to which tie leg is attached



2. The next consists of number of stations into which tie leg is broken -

I 2 FORMAT

MMAXN I-2
No. of stations on tie leg

3. The next group consists of data for the physical properties of the tie leg array segments.

XXTEN(M)	TTDRAG(M)	CABDA(M)	PFDRAG(M)	JMAX(M)
Extensional rigidity (lb.)	Normal drag coeff.	Cable diameter (inches)	Parallel drag coeff.	No. of discrete elements

following each one of these is JMAX(M) cards giving the physical properties of the discrete elements

WEE(J,M)	DRAGCF(J,M)	XXREA(J,M)
Weight (lb.)	Drag coeff.	X-sectional area ft. <sup>2</sup>

Finally the last group of data cards deal with the input values for the current profile.

H(k) F 10.3	V(k) F 10.3
Z coordinate where the velocity profile changes slope (ft.)	The magnitude of current at Z=H(k) (ft./sec.)

COMMON/C19/TIECOM,L  
COMMON/C20/ITEST,JTEST,LTEST,MTEST,LTIE,MCON

C  
C READ AND PRINT INPUT INFORMATION

C  
C CALL INPUT

C  
C COMPUTE MIDSEGMENT DISCRETE ELEMENT KTILDA(M,N)

C  
DO 2 N = 1,3  
MX = MMAX(N)  
DO 2 M = 2,MX  
KX = KMAX(M,N)  
DO 1 K = 1,KX  
IF(SBAR(K,M,N).GT.BLBAR(M,N)/2.)GO TO 2  
1 CONTINUE  
K = KX+1  
2 KTILDA(M,N) = K-1

C  
C SUBSCRIPTS FOR PRIMARY ANCHOR

C  
X(1,1) = AA1(1)  
Y(1,1) = BB1(1)  
Z(1,1) = CC1(1)

C  
C INITIAL VALUES  
LEAP = 1  
JUMP = 1  
PSI = STAPSI  
PI = 3.14159265  
LOOPE = 0  
LOOPA = 0  
ITEST = 0  
JTEST = 0  
MTEST = 0.

C  
C INITIALIZE FORCES FOR TIE LEG POSITION

C  
DO 230 N = 1,3  
MX = MMAX(N)  
DO 230 M = 2,MX  
FFX(M,N) = 0.  
FFY(M,N) = 0.  
230 FFZ(M,N) = 0.  
FFX(L,2) = 0.  
FFY(L,2) = 100.  
FFZ(L,2) = 0.  
FFX(L,3) = 0.  
FFY(L,3) = -100.  
FFZ(L,3) = 0.

C  
C COMPUTE STATION GRAVITY FORCES W(M,N) AND INITIAL FORCES FX,FY,FZ  
CALL GFORC

C  
C COMPUTE CABLE FORCES RX,RY,RZ-TENSION T(M,N) AND STRESSED LENGTH(M,N)

C  
3 DO 7 NN = 1,3  
N = 4-NN  
MX = MMAX(N)



# C. The Computer Program

```

C THIS PROGRAM BY SUBHASH C PAHUJA
C THIS PROGRAM PERFORMS THE MOTION ANALYSIS OF A TRI-MOORED BODY
C STRUCTURE HAVING AN AUXILIARY CABLE BETWEEN TWO CABLE LEGS
C SOLUTION IS BY THE METHOD OF IMAGINARY REACTIONS AND SUCCESSIVE
C APPROXIMATIONS
C THIS PROGRAM UTILIZES THE CONCEPTS OF THE PROGRAM DATUBA WRITTEN BY
C SKOP AND KAPLAN OF THE NAVAL RESEARCH LABORATORY
C NOMENCLATURE FOR THE MAIN STRUCTURE
C N = CABLE INDEX
C M = STATION INDEX
C K = DISCRETE ELEMENT INDEX
C CABLE SEGMENT(M,N)
C BLBAR(M,N) = UNSTRESSED LENGTH
C BL(M,N) = STRESSED LENGTH
C WC(M,N) = WEIGHT/FOOT
C XTEN(M,N) = EXTENSIONAL RIGIDITY
C MU(M,N) = DRAG CHARACTERISTIC
C RD(M,N) = DRAG COEFFICIENT RATIO
C KMAX(M,N) = NO. OF DISCRETE ELEMENTS IN SEGMENT(M,N)
C KTILDA(M,N) = NO. OF DISCRETE ELEMENTS IN FIRST HALF SEGMENT(M,N)
C T(M,N) = TENSION IN SEGMENT(M,N)
C (ALPHA(M,N),BETA(M,N),GAMMA(M,N)) = DIRECTION COSINES
C DISCRETE ELEMENT(K,M,N)
C SBAR(K,M,N) = UNSTRESSED LENGTH FROM STATION (M,N) TO ELEMENT(K,M,N)
C WE(K,M,N) = WEIGHT OF THE ELEMENT
C MUE(K,M,N) = DRAG CHARACTERISTIC
C STATION (M,N)
C (X(M,N),Y(M,N),Z(M,N)) = COORDINATES
C MMAX(N) = NO. OF STATIONS ON CABLE(N)
C (AA1(N),BB1(N),CC1(N)) = ANCHOR COORDINATES
C HORIZL(M,N) = HORIZL DISPLACEMENT FROM THE GRAVITY POSITION
C HEIGHT(M,N) = VERTICAL DISPLACEMENT FROM THE GRAVITY POSITION
C E = ERROR FUNCTION
C COMPE = COMPARISON VALUE FOR E
C TIECOM = COMPARISON VALUE FOR THE FORCE BALANCE CONDITION
C PSI = ANGLE OF ATTACK OF CURRENT
C STAPSI = STARTING VALUE OF THE ANGLE OF ATTACK OF CURRENT
C DELPSI = INCREMENT IN THE VALUE OF THE STAPSI
C ENDPSI = LAST VALUE OF THE ANGLE OF ATTACK OF CURRENT
C REAL BL,BLBAR,BLT,MU,MUE,MUU,MUEE
C COMMON/C1/XF,X(21,3),YF,Y(21,3),ZF,Z(21,3)
C COMMON/C2/FX(21,3),FY(21,3),FZ(21,3)
C COMMON/C3/W(21,3),WC(21,3),WE(10,21,3)
C COMMON/C4/MMAX(3),KMAX(21,3),KTILDA(21,3)
C COMMON/C5/BLBAR(21,3),BL(21,3),SBAR(10,21,3),T(21,3),BLT(21,3)
C COMMON/C6/AA1(3),BB1(3),CC1(3),E,DELTA,JUMP,LOOPE,LOOPA
C COMMON/C7/HORIZL(21,3),HEIGHT(21,3)
C COMMON/C8/CX(21,3),CY(21,3),CZ(21,3)
C COMMON/C9/AV(5),BV(5),VF,V(5),HF,H(5)
C COMMON/C10/COMPE,COMPD,PSI,STAPSI,DELPSI,ENDPSI
C COMMON/C11/XTEN(21,3),MU(21,3),MUE(10,21,3),RD(21,3)
C COMMON/C12/ALPHA(21,3),BETA(21,3),GAMMA(21,3)
C COMMON/C13/RX(21,3),RY(21,3),RZ(21,3)
C COMMON/C14/XO(21,3),YO(21,3),ZO(21,3)
C COMMON/C15/XB(21,3),YB(21,3),ZB(21,3)
C COMMON/C16/FXP(3),FYP(3),FZP(3),XP(3),YP(3),ZP(3)
C COMMON/C17/DELTA1,PDELTA
C COMMON/C18/FFX(21,3),FFY(21,3),FFZ(21,3)

```



```

      GO TO(5,4,4),N
4  RX(MX,N) = FX(MX,N)
   RY(MX,N) = FY(MX,N)
   RZ(MX,N) = FZ(MX,N)
   GO TO 6
5  RX(MX,1) = FX(MX,1)+RX(2,2)+RX(2,3)
   RY(MX,1) = FY(MX,1)+RY(2,2)+RY(2,3)
   RZ(MX,1) = FZ(MX,1)+RZ(2,2)+RZ(2,3)
6  T(MX,N) = SQRT(RX(MX,N)**2+RY(MX,N)**2+RZ(MX,N)**2)
   BL(MX,N) = BLBAR(MX,N)*(1.+T(MX,N)/XTEN(MX,N))
   BLT(MX,N) = BL(MX,N)/T(MX,N)
   MX = MX-1
   DO 7 MM = 2,MX
   M = MX-MM+2
   IF(N.EQ.1)GO TO 8
   IF(M.EQ.1)GO TO 220
8  RX(M,N) = FX(M,N)+RX(M+1,N)
   RY(M,N) = FY(M,N) + RY(M+1,N)
   RZ(M,N) = FZ(M,N) + RZ(M+1,N)
   GO TO 16
220 RX(M,N) = FX(M,N) + RX(M+1,N) + FFX(L,N)
    RY(M,N) = FY(M,N)+RY(M+1,N)+FFY(L,N)
    RZ(M,N) = FZ(M,N)+RZ(M+1,N)+FFZ(L,N)
16  T(M,N) = SQRT(RX(M,N)**2+RY(M,N)**2+RZ(M,N)**2)
    BL(M,N) = BLBAR(M,N)*(1.+T(M,N)/XTEN(M,N))
7  BLT(M,N) = BL(M,N)/T(M,N)

C
C  COMPUTE X,Y,Z COORDINATES OF EACH STATION
C
DO 10 N = 1,3
MX = MMAX(N)
DO 9 M = 2,MX
X(M,N) = BLT(M,N)*RX(M,N)+X(M-1,N)
Y(M,N) = BLT(M,N)*RY(M,N)+Y(M-1,N)
9  Z(M,N) = BLT(M,N)*RZ(M,N)+Z(M-1,N)
GO TO(27,10),N
27 DO 110 NN = 2,3
X(1,NN) = X(MX,1)
Y(1,NN) = Y(MX,1)
110 Z(1,NN) = Z(MX,1)
10 CONTINUE

C
C  COMPUTE ERROR FUNCTION
C
LOOPE = LOOPE+1
E = 0
DO 11 N = 2,3
M = MMAX(N)
11 E = E+((AA1(N)-X(M,N))**2+(BB1(N)-Y(M,N))**2+(CC1(N)-Z(M,N))**2)
IF(E.GT.COMPE)GO TO(19,50,15),LEAP

C
C  UPDATED DIRECTION COSINES
C
DO 300 N = 1,3
MX = MMAX(N)
DO 300 M = 2,MX
ALPHA(M,N) = (X(M,N)-X(M-1,N))/BL(M,N)
BETA(M,N) = (Y(M,N)-Y(M-1,N))/BL(M,N)
300 GAMMA(M,N) = (Z(M,N)-Z(M-1,N))/BL(M,N)

```

```

C      ERROR FUNCTION COMPARISON SATISFIED
      GO TO(51,52),JUMP
C      PRINT AND STORE EQUILIBRIUM POSITION
51 CALL TIELEG(X(L,2),Y(L,2),Z(L,2),X(L,3),Y(L,3),Z(L,3))
      MTEST = MTEST + 1
      POELTA = DELTA
      DELTA = DELTA1
      LEAP = 1
      IF(JTEST.NE.1)GO TO 3
      CALL STAPOS
      JUMP = 2
      LOOPE = 0
      DO 53 N = 1,3
      MX = MMAX(N)
      DO 53 M = 2,MX
      XO(M,N) = X(M,N)
      YO(M,N) = Y(M,N)
      ZO(M,N) = Z(M,N)
      XB(M,N) = X(M,N)
      YB(M,N) = Y(M,N)
      ZB(M,N) = Z(M,N)
53 CONTINUE
      GO TO 61
C
C      COMPARE ACCURACY OF COORDINATES
C
52 DO 55 N = 1,3
MX = MVALUE(N)
DO 55 M = 2,MX
IF(ABS(X(M,N)-XO(M,N)).GT.COMPD.OR.
1ABS(Y(M,N)-YO(M,N)).GT.COMPD.OR.
2ABS(Z(M,N)-ZO(M,N)).GT.COMPD)GO TO 57
55 CONTINUE
C
C      ACCURACY SATISFIED-PRINT EQUILIBRIUM POSITION
C
      ITEST = ITEST+1
      CALL TIELEG(X(L,2),Y(L,2),Z(L,2),X(L,3),Y(L,3),Z(L,3))
      IF(LTEST.NE.200)GO TO 197
C
C
      DO 56 N = 1,3
      MX = MMAX(N)
      DO 56 M = 2,MX
      HORIZL(M,N) = SQRT((X(M,N)-XB(M,N))**2 + (Y(M,N)-YB(M,N))**2)
      HEIGHT(M,N) = Z(M,N)-ZB(M,N)
56 CONTINUE
      CALL DYNPOS
C
      LOOPE = 0
      LOOPA = 0
      LTEST = 100
      GO TO 60
C      ACCURACY NOT ADEQUATE-REITERATE
C
57 DO 59 N = 1,3
MX = MVALUE(N)
DO 59 M = 2,MX
XO(M,N) = X(M,N)

```



```

      YO(M,N) = Y(M,N)
      ZO(M,N) = Z(M,N)
59  CONTINUE
      GO TO 61

C
C  ERROR FUNCTION COMPARISION NOT SATISFIED
C
50  IF(E.LT.EP)GO TO 20
C  INCREASE IN ERROR FUNCTION
      DELTA = DELTA/2.
C
C  COMPUTE ERROR FUNCTION
C
12  DE = DELTA/SQRT(EP)
      DO 13 N = 2,3
      MX = MMAX(N)
      FX(MX,N) = FXP(N)+(AA1(N)-XP(N))*DE
      FY(MX,N) = FYP(N)+(BB1(N)-YP(N))*DE
      FZ(MX,N) = FZP(N)+(CC1(N)-ZP(N))*DE
13  CONTINUE

C
C  CHECK CHANGES IN IMAGINARY REACTIONS
C
      DO 14 N = 2,3
      MX = MMAX(N)
      IF(FX(MX,N).NE.FXP(N).OR.
      IF(FY(MX,N).NE.FYP(N).OR.
      IF(FZ(MX,N).NE.FZP(N))GO TO 3
14  CONTINUE
      LEAP = 3
      GO TO 3
C  NO CHANGE TIME TO QUIT
15  CALL EXITT
      GO TO 100
C  DECREASE IN ERROR FUNCTION
19  LEAP = 2
20  EP = E
      DO 21 N = 2,3
      MX = MMAX(N)
      XP(N) = X(MX,N)
      YP(N) = Y(MX,N)
      ZP(N) = Z(MX,N)
      FXP(N) = FX(MX,N)
      FYP(N) = FY(MX,N)
      FZP(N) = FZ(MX,N)
21  CONTINUE
      GO TO 12

C
C  INCREASE CURRENT ANGLE
C
60  PSI = PSI+DELPSI
      IF(PSI.GE.ENDPSI)GO TO 100
61  COSPSI = COS(PSI*PI/180.)
      SINPSI = SIN(PSI*PI/180.)
      GO TO 62
197 JUMP = 2
      LOOPE = 0
      LOOPA = 0
      GO TO 61

```



```

62 DELTA = DELTA1
   LEAP = 1
   LOOPA = LOOPA+1

```

```

C
C DRAG COEFFICIENTS
C

```

```

DO 30 N = 1, 3
MX = MMAX(N)
DO 30 M = 2, MX
CADELT=SQRT((BETA(M,N)*COSPSI-ALPHA(M,N)*SINPSI)**2
1      + GAMMA(M,N)**2 )
BUFFER = ALPHA(M,N)*COSPSI + BETA(M,N)*SINPSI
CX(M,N)=MU(M,N)*(CADELT*
1      ( (GAMMA(M,N)**2 + BETA(M,N)**2)*COSPSI
2      - ALPHA(M,N) * BETA(M,N) * SINPSI )
3      + RD(M,N) * ALPHA(M,N) * BUFFER )
CY(M,N)=MU(M,N)*(CADELT*
1      ( (GAMMA(M,N)**2 + ALPHA(M,N)**2)*SINPSI
2      - ALPHA(M,N) * BETA(M,N) * COSPSI )
3      + RD(M,N) * BETA(M,N) * BUFFER )
30 CZ(M,N)=MU(M,N)*GAMMA(M,N)*BUFFER*(RD(M,N)-CADELT)

```

```

C
C COMPUTE CABLE DRAG FORCES HX, HY, HZ-AND ELEMENT DRAG FORCES HXE, HYE
C

```

```

DO 40 N = 1, 3
MX = MVALUE(N)
DO 40 M = 2, MX
MXMN=MX-M+1
IF(MXMN-2)23,24,24
24 A1 = AREA(M,N,1)
   A2 = AREA(M+1,N,2)
   HX = CX(M,N) * A1 + CX(M+1,N) * A2
   HY = CY(M,N) * A1 + CY(M+1,N) * A2
   HZ = CZ(M,N) * A1 + CZ(M+1,N) * A2
   DT = AREA(M,N,3)+AREA(M+1,N,4)
   HXE = DT * COSPSI
   HYE = DT * SINPSI
   GO TO 26
23 A1 = AREA(MX,1,1)
   A2 = AREA(2,2,2)
   A3 = AREA(2,3,2)
   HX = CX(M,N) * A1 + CX(2,2) * A2 + CX(2,3) * A3
   HY = CY(M,N) * A1 + CY(2,2) * A2 + CY(2,3) * A3
   HZ = CZ(M,N) * A1 + CZ(2,2) * A2 + CZ(2,3) * A3
   DT = AREA(MX,1,3)+AREA(2,2,4)+AREA(2,3,4)
   HXE = DT * COSPSI
   HYE = DT * SINPSI
C
C NEW TOTAL FORCES
26 FX(M,N) = HX + HXE
   FY(M,N) = HY + HYE
   FZ(M,N) = HZ + W(M,N)
40 CONTINUE
   GO TO 3
100 STOP
   END

```

```

SUBROUTINE TIELEG(X1,Y1,Z1,X2,Y2,Z2)
C SUBROUTINE TIELEG IS USED TO SATISFY THE FOLLOWING CONSTRAINTS
C THE GEOMATRICAL COMPATIBILITY
C THE FORCE BALANCE
C COORDINATES X1,Y1,Z1,X2,Y2,Z2 ARE TRANSFERRED FROM MAIN TO MEET
C THE FIRST CONDITION
C THE SECOND CONDITION IS SATISFIED USING THE SEARCH ROUTINE
C NOMENCLATURE FOT THE TIELEG ARRAY
C M = STATION INDEX
C CABLE SEGMENT(M)
C BAR(M) = UNSTRESSED LENGTH
C B(M) = STRESSED LENGTH
C WWC(M) = WEIGHT/FOOT
C XXTEN(M) = EXTENSIONAL RIGIDITY
C MUU(M) = DRAG CHARACRESTIC
C RRD(M) = DRAG COEFFICIENT RATIO
C JMAX(M) = NO. OF DISCRETE ELEMENTS IN SEGMENT(M)
C JTILDA(M) = NO. OF DISCRETE ELEMENTS IN FIRST HALF SEGMENT(M)
C ST(M) = TENSION IN SEGMENT(M)
C (ALPHA(M),BETA(M),GAMMA(M)) = DIRECTION COSINES
C DISCRETE ELEMENT(J,M)
C SAR(J,M) = UNSTRESSED LENGTH FROM STATION(M) TO ELEMENT(J,M)
C WEE(J,M) = WEIGHT OF THE ELEMENT
C MUUE(J,M) = DRAG CHARATERISTIC
C STATION (M)
C (X(M),Y(M),Z(M)) = COORDINATES
C MMAXN = NO. OF STATIONS ON THE TIELEG ARRAY
C NX(1),Y(1),Z(1) = END COORDINATES FOR THE TIELEG ARRAY
C (X(MX),Y(MX),Z(MX)) = END COORDINATES FOR THE TIELEG ARRAY
C HORIZL(M) = HORIZL DISPLACEMENT FROM THE GRAVITY POSITION
C HEIGHT(M) = VERTICAL DISPLACEMENT FROM THE GRAVITY POSITION
REAL BL,BLBAR,BLT,MU,MUE,MUU,MUUE
DIMENSION XO(22),YO(22),ZO(22),XB(22),YB(22),ZB(22),
1XP(1),YP(1),ZP(1),FXP(1),FYP(1),FZP(1),AA(22),BB(22),CC(22),
2W(22),PREM(22),DISCRT(22)
COMMON/H1/FX(22),FY(22),FZ(22),RX(22),RY(22),RZ(22)
COMMON/H2/RX1(22),RY1(22),RZ1(22)
COMMON/H3/BAR(22),B(22),BT(22),ST(22),XXTEN(22)
COMMON/H4/JMAX(22),JTILDA(22),SAR(5,22)
COMMON/H5/MUU(22),MUUE(5,22),WEE(5,22),WWC(22),RRD(22)
COMMON/H6/ALPHA(22),BETA(22),GAMMA(22)
COMMON/H7/CX(22),CY(22),CZ(22)
COMMON/H8/HORIZL(22),HEIGHT(22)
COMMON/H9/X(22),Y(22),Z(22)
COMMON/H10/E,DELTA,LOOPE,LOOPA
COMMON/H11/MMAXN
COMMON/H12/JUMP,PDIST,DIST
COMMON/C18/FFX(21,3),FFY(21,3),FFZ(21,3)
COMMON/C19/TIECOM,L
COMMON/C9/AV(5),BV(5),VF,V(5),HF,H(5)
COMMON/C10/COMPE,COMPD,PSI,STAPSI,DELPST,ENDPSI
COMMON/C20/ITEST,JTEST,LTEST,MTEST,LTIE,MCON
DATA PI/3.1415926/
X(1) = X1
Y(1) = Y1
Z(1) = Z1
MX = MMAXN
AA(MX) = X2

```



```

BB(MX) = Y2
GG(MX) = Z2
DIST = SQRT((AA(MX)-X(1))**2+(BB(MX)-Y(1))**2+(CC(MX)-Z(1))**2)
IF(IITEST.GE.1)GO TO 61
IF(MTEST.GE.1)GO TO 195
GOMPE = GOMPE/2.
PDIST = DIST

```

```

C
C COMPUTE LENGTH OF SEGEMENT
C

```

```

MX = MMAXN
TMAXN = MX-1
DO 200 M = 2,MX
BAR(M) = DIST/TMAXN
200 CONTINUE

```

```

C
C COMPUTE WEIGHT/FOOT FOR EACH SEGMENT
C

```

```

DO 116 M = 2,MX
PREM(M) = 0.
JX = JMAX(M)
IF(JX.EQ.0)GO TO 117
DO 115 J = 1,JX
PREM(M) = PREM(M) + WEE(J,M)
115 CONTINUE
WWC(M) = PREM(M)/BAR(M)
WWC(M) = -0.99 * WWC(M)
GO TO 116
117 WWC(M) = -0.500
116 CONTINUE

```

```

C
C COMPUTE DISTANCE OF A DISCRETE ELEMENT FROM THE STATION
C

```

```

DO 71 M = 2,MX
JX = JMAX(M)
IF(JX.EQ.0)GO TO 71
SUME = 0.0
DISCRT(M) = BAR(M)/(JX+1)
DO 71 K = 1,JX
SUME = SUME + DISCRT(M)
SAR(K,M) = SUME
71 CONTINUE

```

```

C
C COMPUTE MIDSEGEMENT DISCRETE ELEMENT JTILDA(M)
C

```

```

MX = MMAXN
DO 10 M = 2,MX
KX = JMAX(M)
DO 11 K = 1,KX
IF(SAR(K,M).GT.BAR(M)/2.)GO TO 10
11 CONTINUE
K = KX+1
10 JTILDA(M) = K-1

```

```

C
C COMPUTE STATION GRAVITY FORCES W(M) AND INITIAL FORCES FX,FY,FZ
C

```

```

MX = MMAXN
DO 106 M = 1,MX
FX(M) = 0.

```



```

106 FY(M) = 0.
    FZ(1) = 0.
    WT = 0.
    MX = MMAXN-1
    DO 6 M = 2, MX
        WX = 0.
        KB = JTILDA(M)+1
        KX = JMAX(M)
        IF(KB.GT.KX)GO TO 77
        DO 13 K = KB, KX
13      WX = WX+WEE(K,M)
77      JX = JTILDA(M+1)
        IF(JX.EQ.0)GO TO 78
        DO 2 K = 1, JX
2        WX = WX+WEE(K,M)
78      W(M) = WX + (WWC(M)*BAR(M)/2.) + (WWC(M+1)*BAR(M+1)/2.)
        FZ(M) = W(M)
        WT = WT + W(M)
        MX = MMAXN
        FZ(MX) = (-1.1) * WT
C
C      INITIALIZATION
C
    SQ2 = SQRT(2.)
    DELTA1 = ABS(SQ2*WT)
    LCON = 0
    LTIE = 0
    MCON = 1
195 DELTA = DELTA1
    JUMP = 1
    LEAP = 1
    LOOPA = 0
    LOOPE = 0
C
C      COMPUTE CABLE FORCES RX,RY,RZ - TENSION T(M,N) AND STRESSED LENGTH
C
3    MX = MMAXN
    RX(MX) = FX(MX)
    RY(MX) = FY(MX)
    RZ(MX) = FZ(MX)
    ST(MX) = SQRT(RX(MX)**2+RY(MX)**2+RZ(MX)**2)
    B(MX) = BAR(MX)*(1.+ST(MX)/XXTEN(MX))
    BT(MX) = B(MX)/ST(MX)
    MX = MMAXN-1
    DO 7 MM = 1, MX
        M = MX-MM+1
        GO TO(9,5),M
5      RX(M) = FX(M)+RX(M+1)
        RY(M) = FY(M) + RY(M+1)
        RZ(M) = FZ(M)+RZ(M+1)
        ST(M) = SQRT(RX(M)**2+RY(M)**2+RZ(M)**2)
        B(M) = BAR(M)*(1.+ST(M)/XXTEN(M))
        BT(M) = B(M)/ST(M)
9      RX(M) = FX(M)+RX(M+1)
        RY(M) = FY(M)+RY(M+1)
        RZ(M) = FZ(M)+RZ(M+1)
7    CONTINUE
C
C      COMPUTE X,Y,Z COORDINATES OF EACH STATION

```

C

```

MX = MMAXN
DO 8 M = 2, MX
X(M) = BT(M)*RX(M)+X(M-1)
Y(M) = BT(M)*RY(M)+Y(M-1)
8 Z(M) = BT(M)*RZ(M)+Z(M-1)

```

C

G

C

COMPUTE ERROR FUNCTION

```

LOOPE = LOOPE + 1
E = 0
M = MMAXN
E = E+((AA(M)-X(M))**2+(BB(M)-Y(M))**2+(CC(M)-Z(M))**2)
MX = MMAXN
IF(E.GT.COMPE)GO TO(19,50,15),LEAP

```

C

C

C

UPDATED DIRECTION COSINES

```

MX = MMAXN
DO 300 M = 2, MX
ALPHA(M) = (X(M)-X(M-1))/B(M)
BETA(M) = (Y(M)-Y(M-1))/B(M)
GAMMA(M) = (Z(M)-Z(M-1))/B(M)

```

300 CONTINUE

C

C

C

ERROR FUNCTION COMPARISION SATISFIED

GO TO (51,52), JUMP

51 MX = MMAXN

DO 235 M = 1, MX

RX1(M) = RX(M)

RY1(M) = RY(M)

RZ1(M) = RZ(M)

235 CONTINUE

RX1(1) = -RX1(1)

RY1(1) = -RY1(1)

RZ1(1) = -RZ1(1)

C

C

C

CHECK FOR FORCE BALANCE UNDER GRAVITY FORCES-USE BINARY SEARCH ROUTINE

CALL SEARCH

C

FORCE BALANCE NOT OBTAINED-START AGAIN

IF(LTEST.NE.100)GO TO 100

C

FORCE BALANCE OBTAINED-PRINT AND STORE EQUILIBRIUM POSITION

C

JTEST = JTEST + 1

LTIE = LTIE + 1

CALL STATS

MX = MMAXN

DO 53 M = 2, MX

XO(M) = X(M)

YO(M) = Y(M)

ZO(M) = Z(M)

XB(M) = X(M)

YB(M) = Y(M)

ZB(M) = Z(M)

53 CONTINUE

JUMP = 2

LOOPE = 0



```

      GO TO 100
C
C   COMPARE ACCURACY OF COORDINATES
C
52  MX = MMAXN-1
    DO 55 M = 2, MX
      IF (ABS(X(M)-XO(M)).GT.COMPD.OR.
        1ABS(Y(M)-YO(M)).GT.COMPD.OR.
        2ABS(Z(M)-ZO(M)).GT.COMPD) GO TO 57
    55 CONTINUE
C
C   ACCURACY SATISFIED- PRINT EQUILIBRIUM POSITION FOR HYDRODYNAMIC FORCES
C
      MX = MMAXN
      DO 38 M = 1, MX
        RX1(M) = RX(M)
        RY1(M) = RY(M)
        RZ1(M) = RZ(M)
      38 CONTINUE
      RX1(1) = -RX1(1)
      RY1(1) = -RY1(1)
      RZ1(1) = -RZ1(1)
C
C   CHECK FOR FORCE BALANCE UNDER ACTING FORCES-USE BINARY SEARCH ROUTINE
C
      CALL SEARCH
C   FORCE BALANCE NOT OBTAINED-START AGAIN
      IF (LTEST.NE.200) GO TO 100
C   FORCE BALANCE OBTAIN-PRINT EQUILIBRIUM POSITION FOR
C   HYDRODYNAMIC FORCES
      MX = MMAXN
      DO 56 M = 2, MX
        HORIZL(M) = SQRT((X(M)-XB(M))**2 + (Y(M)-YB(M))**2)
        HEIGHT(M) = Z(M)-ZB(M)
      56 CONTINUE
      CALL DYNAMS
      GO TO 100
C   ACCURACY NOT ADEQUATE - REITERATE
57  MX = MMAXN-1
    DO 37 M = 2, MX
      XO(M) = X(M)
      YO(M) = Y(M)
      ZO(M) = Z(M)
    37 CONTINUE
    GO TO 62
C
C   TEST IF PRECISION FOCUS SHOULD BE APPLIED
C
50  IF (ABS(AA(MX)-X(MX)).GE.DOMPE.OR.ABS(CC(MX)-Z(MX)).GE.
      1DOMPE) GO TO 734
      IF (YP(1).EQ.Y(MX)) GO TO 734
      DELFYP = (FY(MX)-FYP(1))/(Y(MX)-YP(1))* (BB(MX)-Y(MX))
      FYP(1) = FY(MX)
      YP(1) = Y(MX)
      FY(MX) = FY(MX)+DELFYP
      GO TO 3
C
C   ERROR FUNCTION COMPARISON NOT SATISFIED
C

```



```

734 IF(E.LT.EP)GO TO 20
C INCREASE IN ERROR FUNCTION
  DELTA = DELTA/2.
G COMPUTE IMAGINARY REACTIONS
  12 DE = DELTA/SQRT(EP)
  MX = MMAXN
  FX(MX) = FXP(1)+(AA(MX)-XP(1))*DE
  FY(MX) = FYP(1)+(BB(MX)-YP(1))*DE
  FZ(MX) = FZP(1)+(CC(MX)-ZP(1))*DE
C CHECK CHANGES IN IMAGINARY REACTIONS
  IF(FX(MX).NE.FXP(1).OR.FY(MX).NE.FYP(1).OR.
  1 FZ(MX).NE.FZP(1))GO TO 3
C NO CHANGE IN FORCE TIME TO QUIT
  LEAP = 3
  GO TO 3
  15 IF(NTTEST.GE.1)GO TO 150
  FFY(L,2) = 2.0 * FFY(L,2)
  FFY(L,3) = 2.0 * FFY(L,3)
  GO TO 100
150 MCON = 2
162 CALL SEARCH
  GO TO 100
C DECREASE IN ERROR FUNCTION
  19 LEAP = 2
  20 EP = E
  MX = MMAXN
  XP(1) = X(MX)
  YP(1) = Y(MX)
  ZP(1) = Z(MX)
  FXP(1) = FX(MX)
  FYP(1) = FY(MX)
  FZP(1) = FZ(MX)
  GO TO 12
C ADD HYDRODYNAMIC FORCE TO THE SYSTEM
  61 COSPSI = COS(PSI*PI/180.)
  SINPSI = SIN(PSI*PI/180.)
  62 DELTA = DELTA1
  LEAP = 1
  LOOPA = LOOPA + 1
C DRAG COEFFICIENTS
  MX = MMAXN
  DO 31 M = 2,MX
  CAPDEL = SQRT((BETA(M)*COSPSI-ALPHA(M)*SINPSI)**2+GAMMA(M)**2)
  BUFFER = ALPHA(M)*COSPSI + BETA(M)*SINPSI
  CX(M) = MUU(M)*(CAPDEL*(GAMMA(M)**2+BETA(M)**2)*COSPSI-
  1 ALPHA(M)*BETA(M)*SINPSI+RRD(M)*BETA(M)*BUFFER)
  CY(M) = MUU(M)*(CAPDEL*(GAMMA(M)**2+ALPHA(M)**2)*SINPSI-
  1 ALPHA(M)*BETA(M)*COSPSI+RRD(M)*BETA(M)*BUFFER)
  CZ(M) = MUU(M)*GAMMA(M)*BUFFER*(RRD(M)-CAPDEL)
  31 CONTINUE
  MX = MMAXN-1
  DO 41 M = 2,MX
  A1 = TAREA(M,1)
  A2 = TAREA(M+1,2)
  HX = CX(M)*A1+CX(M+1)*A2
  HY = CY(M)*A1+CY(M+1)*A2
  HZ = CZ(M)*A1+CZ(M+1)*A2
  DT = TAREA(M,3) + TAREA(M+1,4)
  HXE = DT*COSPSI

```

# SUBROUTINE INPUT

```

REAL BL,BLBAR,BLT,MU,MUE,MUU,MUUE
COMMON/C3/W(21,3),WG(21,3),WE(10,21,3)
COMMON/C4/MMAX(3),KMAX(21,3),KTILDA(21,3)
COMMON/C5/BLBAR(21,3),BL(21,3),SBAR(10,21,3),T(21,3),BLT(21,3)
COMMON/C6/AA1(3),BB1(3),CC1(3),E,DELTA,JUMP,LOOPE,LOOPA
COMMON/C9/AV(5),BV(5),VF,V(5),HF,H(5)
COMMON/C10/COMPE,COMP,PSI,STAPSI,DELPSI,ENDPSI
COMMON/C11/XTEN(21,3),MU(21,3),MUE(10,21,3),RD(21,3)
COMMON/H3/BAR(22),B(22),BT(22),ST(22),XXTEN(22)
COMMON/H4/JMAX(22),JTILDA(22),SAR(5,22)
COMMON/H5/MUU(22),MUUE(5,22),WEE(5,22),WWC(22),RRD(22)
COMMON/H11/MMAXN
COMMON/C19/TIECOM,L

```

C

## C COMPARISON VALUES AND CURRENT ANGLE REQUIREMENTS

```

WRITE(6,101)
101 FORMAT('1',15X,'INPUT DATA COMMON TO,BOTH,THE TIE LEG AND THE
1TRI MOORED STRUCTURE'/)
WRITE(6,35)
35 FORMAT(5X,'COMPARISON VALUES AND CURRENT ANGLE REQUIREMENTS'/)
READ(5,1)COMPE,COMP,STAPSI,DELPSI,ENDPSI,TIECOM
1 FORMAT(F10.2,5F10.0)
WRITE(6,30)COMPE,COMP,STAPSI,DELPSI,ENDPSI,TIECOM
30 FORMAT(15X,'COMPARISON VALUE FOR E =',E15.6/15X,'COMPARISON VAL
1UE FOR DISPLACEMENT =',E15.6/15X,'INITIAL CURRENT ANGLE =',F8.3,
2'DEGREES'/15X,'INCREMENT OF ANGLE =',F8.3,'DEGREES'/15X,'FINAL CUR
3RENT ANGLE =',F8.3,'DEGREES'/15X,'COMPARISON VALUE FOR THE TIE LEG
4JOINT =',E16.5/)

```

C

```

WRITE(6,102)
102 FORMAT('///15X,' INPUT DATA FOR THE TRI MOORED STRUCTURE'//)

```

C

## C ANCHOR POSITIONS

```

WRITE(6,40)
40 FORMAT(10X,'ANCHOR POSITIONS'/)
DO 31 N = 1,3
31 READ(5,2)AA1(N),BB1(N),CC1(N)
2 FORMAT(3F10.0)

```

## C NO. OF STATIONS PER CABLE

```

WRITE(6,45)(AA1(N),BB1(N),CC1(N),N=1,3)
45 FORMAT(2X,'AA1=',F10.0,'BB1=',F10.0,'CC1=',F10.0/)
WRITE(6,50)
50 FORMAT(10X,'NO. OF STATIONS PER CABLE'/)
READ(5,3)(MMAX(N),N=1,3)

```

```

3 FORMAT(3I5)
WRITE(6,55)(N,MMAX(N),N=1,3)

```

```

55 FORMAT(2X,'CABLE NO.',I1,I5)
READ(5,36)IL

```

```

36 FORMAT(I2)
WRITE(6,60)

```

```

60 FORMAT(/,5X,'PROPERTIES OF SEGMENTS AND DISCRETE ELEMENTS'/)
DO 10 N=1,3
WRITE(6,70)N

```

```

70 FORMAT(60X,'CABLE NO.',I1/)
WRITE(6,65)

```

```

65 FORMAT(4X,'M',3X,'STRESSED LEN.',3X,'WEIGHT/FT',3X,'EXT.RIGIDIT
1Y',2X,'DRAG CHARACTER',1X,'DRAG COE. RATIO',2X,'K',

```



HYE = DT\*SINPSI  
26 FX(M) = HX + HXE  
FY(M) = HY + HYE  
FZ(M) = HZ + W(M)  
41 CONTINUE  
GO TO 3  
100 RETURN  
END



```

2 6X,'SBAR',9X,'WEIGHT',11X,'MUE')
MMN=MMAX(N)
DO 10 M=2,MMN
  READ(5,15)BLBAR(M,N),WC(M,N),TDRAG,CABDIA,PDRAG,XTEN(M,N),
  1KMAX(M,N)
15 FORMAT(5F10.3,F10.2,I2)
  MU(M,N)=1.94*TDRAG*CABDIA/24.
  RD(M,N)=PDRAG/TDRAG
  I=M-1
  WRITE(6,24)I,BLBAR(M,N),WC(M,N),XTEN(M,N),MU(M,N),RD(M,N),
  1KMAX(M,N)
24 FORMAT(15,5E15.6,I5)
  IF(KMAX(M,N).EQ.0) GO TO 10
  KKN=KMAX(M,N)
  DO 5 K=1,KKN
  READ(5,20)SBAR(K,M,N),WE(K,M,N),DRAGCF,XAREA
20 FORMAT(4F10.2)
  MUE(K,M,N)=1.94*DRAGCF*XAREA/2.
  5 WRITE(6,25)SBAR(K,M,N),WE(K,M,N),MUE(K,M,N)
25 FORMAT(85X,3E15.6)
10 CONTINUE

C
C   INPUT DATA FOR TIELEG
C
  WRITE(6,103)
103 FORMAT('1',15X,'INPUT DATA FOR TIE LEG'//)
  WRITE(6,37)I
37 FORMAT(10X,'TIE LEG JOINS CABLE AT STATION NO.',I2/)
  READ(5,74)MMAXN
74 FORMAT(13)
  WRITE(6,72)MMAXN
72 FORMAT(5X,'NO. OF STATIONS ON THE TIELEG=',I3)
  WRITE(6,104)
104 FORMAT(4X,'M',3X,'EXT.RIGIDITY',3X,'DRAG CHARACTER',2X,'DRAG CCE-
  1ATIO',3X,'J',20X,'WEIGHT',10X,'MUUE'//)
  MX = MMAXN
  DO 17 M = 2,MX
  READ(5,14)XXTEN(M),TTDRAG,CABDA,PPDRAG,JMAX(M)
14 FORMAT(4F10.3,I2)
  MUU(M) = 1.94*TTDRAG*CABDA/24.
  RRD(M) = PPDRAG/TTDRAG
  I = M-1
  WRITE(6,16)I,XXTEN(M),MUU(M),RRD(M),JMAX(M)
16 FORMAT(14,3E16.7,I4)
  IF(JMAX(M).EQ.0)GO TO 17
  LKN = JMAX(M)
  DO 18 K = 1,LKN
  READ(5,29)WEE(K,M),GRAGCF,XXAREA
29 FORMAT(3F10.2)
  MUUE(K,M) = 1.94*GRAGCF*XXAREA/2.
18 WRITE(6,21)WEE(K,M),MUUE(K,M)
21 FORMAT(75X,2E15.6)
17 CONTINUE
C   PROVIDE VELOCITY PROFILE IN SUBROUTINE VPROFILE
  CALL VPROFL
  WRITE(6,75)
75 FORMAT(40X,'XXXXXXXXXXXXXXXXXXXXXXXXXXXX',T1,'1'//)
  RETURN
  END

```

# SUBROUTINE SEARCH

```

C THIS SUBROUTINE INVOLVES THE CONCEPTS OF BINARY SEARCH
C A NEW VALUE OF THE FORCES IS FOUND THAT ALWAYS LIES INBETWEEN
C THE TWO PREVIOUS VALUES
C THE CONCEPT IS SIMILIAR TO THE DAMPING FORCE AND AS SUCH BRINGS ABOUT
C FAST CONVERGENCE
C
C DIMENSION ZXP(2),ZYP(2),ZZP(2)
COMMON/C4/MMAX(3),KMAX(21,3),KTILDA(21,3)
COMMON/C18/FFX(21,3),FFY(21,3),FFZ(21,3)
COMMON/C19/TIECOM,L
COMMON/H11/MMAXN
COMMON/H2/RX1(22),RY1(22),RZ1(22)
COMMON/C20/ITEST,JTEST,LTEST,MTEST,LTIE,MCON
GO TO(55,156),MCON
55 ZXP(1) = FFX(L,2)
ZYP(1) = FFY(L,2)
ZZP(1) = FFZ(L,2)
ZXP(2) = FFX(L,3)
ZYP(2) = FFY(L,3)
ZZP(2) = FFZ(L,3)
C TEST FOR FORCE BALANCE
MX = MMAXN
IF((FFX(L,2)+RX1(1)).GT.TIECOM.OR.
1 (FFY(L,2)+RY1(1)).GT.TIECOM.OR.
2 (FFZ(L,2)+RZ1(1)).GT.TIECOM)GO TO 77
76 IF((FFX(L,3)+RX1(MX)).GT.TIECOM.OR.
1 (FFY(L,3)+RY1(MX)).GT.TIECOM.OR.
2 (FFZ(L,3)+RZ1(MX)).GT.TIECOM)GO TO 77
IF(LTIE.GE.1)GO TO 90
C FORCE BALANCE UNDER GRAVITY FORCES OBTAINED-RETURN AND PRINT RESULTS
LTEST = 100
GO TO 100
C FORCE BALANCE UNDER ACTING FORCES OBTAINED-RETURN AND PRINT RESULTS
90 LTEST = 200
GO TO 100
156 MX = MMAXN
RY1(1) = FFY(L,2)
RY1(MX) = FFY(L,3)
C FORCE BALANCE NOT OBTAINED-USE BINARY SEARCH TO OBTAIN NEW VALUES
C AND START ALL OVER AGAIN
77 FFX(L,2) = -RX1(1)
XNO = ABS(RY1(1))
YNO = ABS(ZYP(1))
HNO = AMIN1(XNO,YNO)
ZNO = ABS(XNO-YNO)
FFY(L,2) = (ZNO/2. + HNO)
FFZ(L,2) = -RZ1(1)
FFX(L,3) = -RX1(MX)
X1NO = ABS(RY1(MX))
Y1NO = ABS(ZYP(2))
H1NO = AMIN1(X1NO,Y1NO)
Z1NO = ABS(X1NO-Y1NO)
FFY(L,3) = -(Z1NO/2. + H1NO)
FFZ(L,3) = -RZ1(MX)
100 RETURN
END

```



```

SUBROUTINE GFORC
REAL BL,BLBAR,BLT,MU,MUE,MUU,MUUE
COMMON/C2/FX(21,3),FY(21,3),FZ(21,3)
COMMON/C3/W(21,3),WC(21,3),WE(10,21,3)
COMMON/C4/MMAX(3),KMAX(21,3),KTILDA(21,3)
COMMON/C5/BLBAR(21,3),BL(21,3),SBAR(10,21,3),T(21,3),BLT(21,3)
COMMON/C6/AA1(3),BB1(3),CC1(3),E,DELTA,JUMP,LOOPE,LOOPA
COMMON/C17/DELTA1,PDELTA
C COMPUTE STATION GRAVITY FORCES W(M,N) AND INITIAL FORCES FX,FY,FZ
WT = 0.
DO 3 N = 1, 3
MX = MMAX(N) - 1
DO 3 M = 2, MX
WX = 0.
FX(M,N) = 0.
FY(M,N) = 0.
KB = KTILDA(M,N) + 1
KX = KMAX(M,N)
IF(KB.GT.KX) GO TO 10
DO 1 K = KB, KX
1 WX = WX + WE(K,M,N)
10 KX = KTILDA(M+1,N)
IF(KX.EQ.0) GO TO 30
DO 2 K = 1, KX
2 WX = WX + WE(K,M+1,N)
30 W(M,N) = WX+WC(M,N)*BLBAR(M,N)/2.+WC(M+1,N)*BLBAR(M+1,N)/2.
FZ(M,N) = W(M,N)
3 WT = WT + W(M,N)
WX = 0.
FX(MMAX(1),1) = 0.
FY(MMAX(1),1) = 0.
KB = KTILDA(MMAX(1),1) + 1
KX = KMAX(MMAX(1),1)
IF(KB.GT.KX) GO TO 20
DO 4 K = KB, KX
4 WX = WX + WE(K,MMAX(1),1)
20 CONTINUE
DO 6 N = 2, 3
KX = KTILDA(2,N)
FX(MMAX(N),N) = 0.
FY(MMAX(N),N) = 0.
IF(KX.EQ.0) GO TO 6
DO 5 K = 1, KX
5 WX = WX + WE(K,2,N)
6 WX = WX+WC(2,N)*BLBAR(2,N)/2.
W(MMAX(1),1) = WX+WC(MMAX(1),1)*BLBAR(MMAX(1),1)/2.
FZ(MMAX(1),1) = W(MMAX(1),1)
DO 35 N = 2,3
FX(1,N) = FX(MMAX(1),1)
FY(1,N) = FY(MMAX(1),1)
35 FZ(1,N) = FZ(MMAX(1),1)
WT = WT + W(MMAX(1),1)
DO 7 N = 2, 3
7 FZ(MMAX(N),N) = -WT / 3.
C
C COMPUTE INITIAL DELTA
SQ2 = SQRT (.2.)
DELTA1 = ABS(SQ2*WT/3.)

```



DELTA = DELTA1  
RETURN  
END

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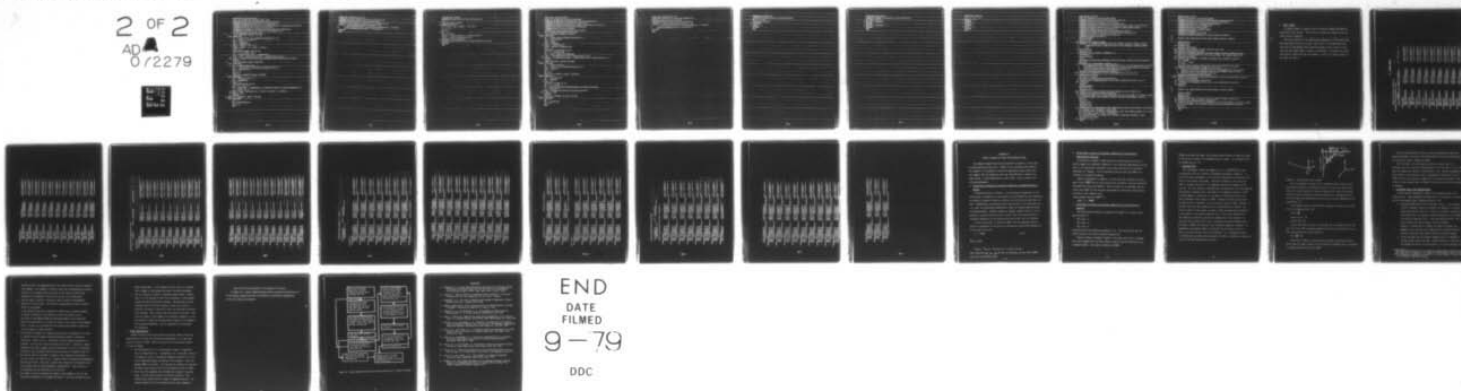
NEW HAMPSHIRE UNIV DURHAM ENGINEERING DESIGN AND ANA--ETC F/G 13/10  
A COMPUTER ANALYSIS OF A TRI-MOORED BUOY STRUCTURE WITH AN INTE--ETC(U)  
MAY 72 S C PAHUJA, R W CORELL

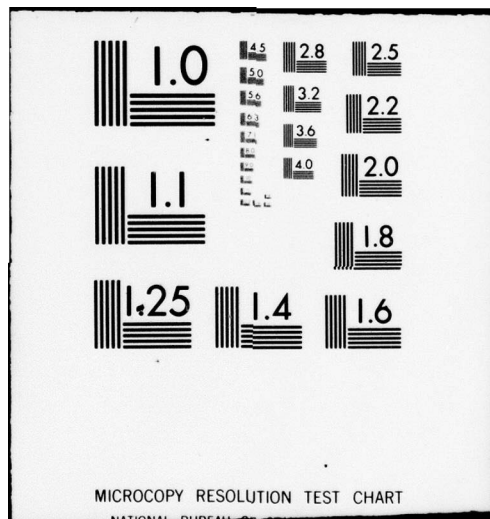
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```

FUNCTION AREA(M,N,IGO)
REAL BL,BLBAR,BLT,MU,MUE,MUU,MUUE
COMMON/C1/XF,X(21,3),YF,Y(21,3),ZF,Z(21,3)
COMMON/C4/MMAX(3),KMAX(21,3),KTILDA(21,3)
COMMON/C5/BLBAR(21,3),BL(21,3),SBAR(10,21,3),T(21,3),BLT(21,3)
COMMON/C9/AV(5),BV(5),VF,V(5),HF,H(5)
COMMON/C11/XTEN(21,3),MU(21,3),MUE(10,21,3),RD(21,3)
COMMON/C12/ALPHA(21,3),BETA(21,3),GAMMA(21,3)
GO TO(100,200,300,400),IGO
C LINE INTEGRAL BELOW STATION
100 CONTINUE
KL = LIMIT(Z(M-1,N)+GAMMA(M,N)*BL(M,N)/2.)
KU = LIMIT ( Z(M,N) )
TOP = BL(M,N)
BOT = BL(M,N)/2.
2 KMIN = MINO ( KU, KL )
KTOP = KMIN - 1 + IABS ( KU-KL )
SUM = 0.
IF(KMIN.GT.KTOP) GO TO 10
DO 1 K = KMIN, KTOP
XI = (H(K)-Z(M-1,N)) / GAMMA(M,N)
1 SUM = SUM + AREAS(M,N,K,XI) - AREAS(M,N,K+1,XI)
10 AREA = SYGN(KU-KL)*SUM+AREAS(M,N,KU,TOP)-AREAS(M,N,KL,BOT)
RETURN
C LINE INTEGRAL ABOVE STATION
200 CONTINUE
KL = LIMIT(Z(M-1,N))
KU = LIMIT(Z(M-1,N)+GAMMA(M,N)*BL(M,N)/2.)
TOP = BL(M,N)/2.
BOT = 0.
GO TO 2
C DRAG OF ELEMENTS BELOW STATION
300 CONTINUE
KB = KTILDA(M,N) + 1
KX = KMAX(M,N)
5 SUM = 0.
IF(KB.GT.KX)GO TO 40
DO 4 K = KB, KX
ZT = Z(M-1,N) + GAMMA(M,N) * SBAR(K,M,N)*(1.+T(M,N)/XTEN(M,N))
J = LIMIT( ZT )
4 SUM = SUM + MUE(K,M,N) * (AV(J) + BV(J) * ZT)**2
40 AREA = SUM
RETURN
C DRAG OF ELEMENTS ABOVE STATION
400 CONTINUE
KB = 1
KX = KTILDA(M,N)
GO TO 5
END

```

```

FUNCTION AREAS(M,N,K,XI)
COMMON/C1/XF,X(21,3),YF,Y(21,3),ZF,Z(21,3)
COMMON/C9/AV(5),BV(5),VF,V(5),HF,H(5)
COMMON/C12/ALPHA(21,3),BETA(21,3),GAMMA(21,3)
AREAS = (AV(K)+BV(K)*Z(M-1,N))**2 * XI
1      +(AV(K)+BV(K)*Z(M-1,N))*BV(K)*GAMMA(M,N) * (XI**2)
2      +(BV(K)*GAMMA(M,N))**2 * (XI**3/3.)
RETURN
END

```



```

SUBROUTINE VPROFL
COMMON/C9/AV(5),BV(5),VF,V(5),HF,H(5)
K=1
1 READ(5,2)H(K),V(K)
2 FORMAT(2F10.0)
IF ( H(K) .GE. 40000. ) GO TO 3
K = K + 1
GO TO 1
3 KX = K
DO 4 K=2,KX
BV(K) = (V(K)-V(K-1)) / (H(K)-H(K-1))
AV(K) = V(K-1)-BV(K)*H(K-1)
4 WRITE(6,9)AV(K),BV(K)
9 FORMAT(10X,6HAV(K)=,E16.5/10X,6HBV(K)=,E16.5)
RETURN
END

```



```

FUNCTION TAREA(M,MGO)
REAL BL,BLBAR,BLT,MU,MUE,MUU,MUUE
COMMON/H3/BAR(22),B(22),BT(22),ST(22),XXTEN(22)
COMMON/H4/JMAX(22),JTILDA(22),SAR(5,22)
COMMON/H5/MUU(22),MUUE(5,22),WEE(5,22),WWG(22),RRD(22)
COMMON/H6/ALPHA(22),BETA(22),GAMMA(22)
COMMON/H9/X(22),Y(22),Z(22)
COMMON/C9/AV(5),BV(5),VF,V(5),HF,H(5)
GO TO(1000,2000,3000,4000),MGO
C
LINE INTEGRAL ABOVE STATION
1000 CONTINUE
KL = LIMIT(Z(M-1)+GAMMA(M)*B(M)/2.)
KU = LIMIT(Z(M))
TOP = B(M)
BOT = B(M)/2.
2 KMIN = MINO(KU,KL)
KTOP = KMIN-1+IABS(KU-KL)
SUM = 0.
IF(KMIN.GT.KTOP)GO TO 10
DO 1 K = KMIN,KTOP
XI = (H(K)-Z(M-1))/GAMMA(M)
1 SUM = SUM+TAREAS(M,K,XI)-TAREAS(M,K+1,XI)
10 TAREA = SYGN(KU-KL)*SUM + TAREAS(M,KU,TOP)-TAREAS(M,KL,BOT)
RETURN
C
LINE INTEGRAL BELOW STATION
2000 CONTINUE
KL = LIMIT(Z(M-1))
KU = LIMIT(Z(M-1)+GAMMA(M)*B(M)/2.)
TOP = B(M)/2.
BOT = 0.
GO TO 2
C
DRAG OF ELEMENTS ABOVE STATIONS
3000 CONTINUE
KB = JTILDA(M)+1
KX = JMAX(M)
5 SUM = 0.
IF(KB.GT.KX)GO TO 20
DO 4 K = KB,KX
ZT = Z(M-1)+GAMMA(M)*SAR(K,M)*(1.+ST(M)/XXTEN(M))
J = LIMIT(ZT)
4 SUM = SUM+MUUE(K,M)*(AV(J)+BV(J)*ZT)**2
20 TAREA = SUM
RETURN
C
DRAG OF ELEMENTS BELOW STATION
4000 CONTINUE
KB = 1
KX = JTILDA(M)
GO TO 5
END

```

```

FUNCTION TAREAS(M,K,XI)
COMMON/H6/ALPHA(22),BETA(22),GAMMA(22)
COMMON/H9/X(22),Y(22),Z(22)
COMMON/C9/AV(5),BV(5),VF,V(5),HF,H(5)
TAREAS = (AV(K)+BV(K)*Z(M-1))**2 * XI
1      +(AV(K)+BV(K)*Z(M-1))*BV(K)*GAMMA(M) * (XI**2)
3      +(BV(K)*GAMMA(M))**2 * (XI**3/3.)
RETURN
END

```



FUNCTION LIMIT(ZT)

COMMON/C9/AV(5),BV(5),VF,V(5),HF,H(5)

DO 1 J = 2,5

IF(ZT-H(J))2,2,1

1 CONTINUE

2 LIMIT = J

RETURN

END



```
FUNCTION MVALUE(N)  
COMMON/C4/MMAX(3),KMAX(21,3),KTILDA(21,3)  
GO TO(1,2,2),N  
1 MVALUE = MMAX(N)  
RETURN  
2 MVALUE = MMAX(N) - 1  
RETURN  
END
```

```
FUNCTION SYGN(J)  
IF ( J ) 1, 2, 3  
1 SYGN = -1.  
  RETURN  
2 SYGN = 0  
  RETURN  
3 SYGN = 1.  
  RETURN  
END
```



```

SUBROUTINE OUTPUT
REAL BL, BLBAR, BLT, MU, MUE, MUU, MUUE
COMMON/C1/XF, X(21,3), YF, Y(21,3), ZF, Z(21,3)
COMMON/C2/FX(21,3), FY(21,3), FZ(21,3)
COMMON/C4/MMAX(3), KMAX(21,3), KTILDA(21,3)
COMMON/C5/BLBAR(21,3), BL(21,3), SBAR(10,21,3), T(21,3), BLT(21,3)
COMMON/C6/AA1(3), BB1(3), CC1(3), E, DELTA, JUMP, LOOPE, LOOPA
COMMON/C7/HORIZL(21,3), HEIGHT(21,3)
COMMON/C10/COMPE, COMPU, PSI, STAPSI, DELPSI, ENOPSI
COMMON/C11/XTEN(21,3), MU(21,3), MUE(10,21,3), RD(21,3)
COMMON/C13/RX(21,3), RY(21,3), RZ(21,3)
COMMON/C17/DELTA1, PDELTA
ENTRY STAPOS
WRITE(6,13)
16 WRITE(6,5)E, PDELTA, LOOPE
5 FORMAT(1X,41EQUILIBRIUM POSITION UNDER GRAVITY FORCES, //10X,
1 2HE=, E16.9, 10X, 6HDELTA=, E16.9, 10X, 19HNO. OF ERROR LOOPS=
2 ,15)
6 DO 10 N=1,3
WRITE(6,7)N
7 FORMAT(//,3X,13HCABLE NUMBER=,12)
MX=MMAX(N)
DO 10 M=2,MX
I=M-1
WRITE(6,8)I, FX(M,N), FY(M,N), FZ(M,N), X(M,N), Y(M,N), Z(M,N), T(M,N),
1BL(M,N)
8 FORMAT(//,5X,15HSEGMENT NUMBER=,12, /
1 10X,8HFX(M,N)=, E16.9, 10X, 8HFY(M,N)=, E16.9, 10X, 8HFZ(M,N)=, E16.9/
2 10X,7HX(M,N)=, E16.9, 10X, 7HY(M,N)=, E16.9, 10X, 7HZ(M,N)=, E16.9/
3 10X,8H T(M,N)=, E16.9, 10X, 8HBL(M,N)=, E16.9)
WRITE(6,639)RX(M,N), RY(M,N), RZ(M,N)
639 FORMAT(10X,8HRX(M,N)=, E16.9, 10X, 8HRY(M,N)=, E16.9, 10X,
18HRZ(M,N)=, E16.9)
GO TO(10,9), JUMP
9 WRITE(6,11)HORIZL(M,N), HEIGHT(M,N)
11 FORMAT(15X,12HHORIZL(M,N)=, E16.9, 10X, 12HHEIGHT(M,N)=, E16.9)
10 CONTINUE
RETURN
ENTRY DYNPOS
WRITE(6,13)
WRITE(6,116)
116 FORMAT(13X,'EQUILIBRIUM POSITION UNDER ACTING FORCES'//)
17 WRITE(6,12)E, DELTA, LOOPE, LOOPA, PSI
12 FORMAT(10X,2HE=, E16.9, 10X, 6HDELTA=, E16.9/13X,19HNO. OF ERROR LOOPS
1=,15, 10X, 22HNO. OF ACCURACY LOOPS=,15/15X,14HCURRENT ANGLE=,
2F8.3/)
GO TO 6
ENTRY EXITT
WRITE(6,13)
WRITE(6,14)
14 FORMAT(1X//1X,90HPROBLEM NOT COMPLETED, DELTA HAS GOTTEN TOO SMALL
1TO CHANGE THE IMAGINARY REACTIONS
2/1X,100HEITHER ACCURACY REQUIREMENTS ARE TOO SMALL(COMPE) OR A CAB
3LE HAS GONE SLACK (CHECK TENSIONS).
4/1X,55HPRINTOUT IS GIVEN FOR TROUBLE SHOOTING PURPOSES ONLY. )
GO TO(16,17), JUMP
13 FORMAT(14H1)
END

```



# SUBROUTINE TIEOUT

REAL BL, BLBAR, BLT, MU, MUE, MUU, MUUE

COMMON/H1/FX(22), FY(22), FZ(22), RX(22), RY(22), RZ(22)

COMMON/H3/BAR(22), B(22), BT(22), ST(22), XTEN(22)

COMMON/H4/JMAX(22), JTILDA(22), SAR(5,22)

COMMON/H8/HORIZL(22), HEIGHT(22)

COMMON/H9/X(22), Y(22), Z(22)

COMMON/H10/E, DELTA, LOOPE, LOOPA

COMMON/H11/MAXN

COMMON/H12/JUMP, PDIST, DIST

COMMON/G10/COMPE, COMPD, PSI, STAPSI, DELPSI, ENDPsi

C  
C  
C

OUTPUT FOR EQUILIBRIUM POSITION UNDER GRAVITY FORCES

## ENTRY STATS

WRITE(6,13)

13 FORMAT(1H1)

WRITE(6,100)PDIST

100 FORMAT(10X, 'LENGTH OF THE TIELEG=' ,E16.9/)

26 WRITE(6,25)E, DELTA, LOOPE

25 FORMAT(1X41'EQUILIBRIUM POSITION UNDER GRAVITY FORCES'//10X,

12HE=,F16.9,10X,6HDELTA=,E16.9,10X,19HNO. OF ERROR LOOPS=,15)

27 WRITE(6,101)DIST

101 FORMAT(10X, 'CHORDAL DISTANCE BETWEEN END POINTS=' ,E16.9/)

MX = MAXN

DO 20 M = 2, MX

I = M-1

WRITE(6,28)I, FX(M), FY(M), FZ(M), X(M), Y(M), Z(M), ST(M), B(M)

28 FORMAT(/,5X, 'SEGMENT NUMBER =' ,12, /10X,

1 6HFX(M)=,E16.9,10X,6HFY(M)=,E16.9,10X,6HFZ(M)=,E16.9, /10X,

2 5HX(M)=,E16.9,10X,5HY(M)=,E16.9,10X,5HZ(M)=,E16.9, /10X,

3 6HST(M)=,E16.9,10X,5HB(M)=,E16.9)

WRITE(6,1000)RX(M), RY(M), RZ(M)

1000 FORMAT(10X,6HRX(M)=,E16.9,10X,6HRY(M)=,E16.9,10X,6HRZ(M)=,E16.9)

GO TO (20,29), JUMP

29 WRITE(6,11)HORIZL(M), HEIGHT(M)

11 FORMAT(15X,10HHORIZL(M)=,E16.9,10X,10HHEIGHT(M)=,E16.9)

20 CONTINUE

RETURN

C  
C  
C

OUTPUT FOR EQUILIBRIUM POSITION UNDER ACTING FORCES

## ENTRY DYNAMS

WRITE(6,13)

WRITE(6,16)

16 FORMAT(10X, 'EQUILIBRIUM POSITION UNDER ACTING FORCES'//)

17 WRITE(6,12)E, DELTA, LOOPE, LOOPA, PSI

12 FORMAT(10X,2HE=,E16.9,10X,6HDELTA=,E16.9/13X,19HNO. OF ERROR LOOPS

1=,15,10X,22HNO. OF ACCURACY LOOPS=,15/15X,14HCURRENT ANGLE=,

2F8.3/)

GO TO 27

END

#### D. SAMPLE OUTPUT

A computer OUTPUT as obtained from the subroutine OUTPUT and TIEOUT is reproduced in this section. This is done to present the format in which the output should be expected.

First part consists of the equilibrium configuration of the array system when only gravity forces are acting. Next follows the configuration under the action of hydrodynamic forces produced because of the current (in this case only zero degree angle of attack is considered). The printout for main cable array is shown only for cable number 1. There is a similar format for the other two cables.

# D. Sample Output

LENGTH OF THE TIELE= 0.306128047E 05

EQUILIBRIUM POSITION UNDER GRAVITY FORCES

E= 0.026809946 DELTA= 0.477388382E 01 NO. OF ERROR LOOPS= 29  
 CHORDAL DISTANCE BETWEEN END POINTS= 0.306128047E 05

SEGMENT NUMBER= 1

FX(M)= 0.0  
 FY(M)= 0.0  
 X(M)= 0.883713672E 04  
 Y(M)= 0.106570386E 04  
 ST(M)= 0.0  
 RM(M)= 0.0

FY(M)= 0.0  
 V(M)= -0.137770117E 05  
 B(M)= 0.153135815E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.813600159E 02  
 RZ(M)= 0.540116272E 02

SEGMENT NUMBER= 2

FX(M)= 0.0  
 FY(M)= 0.0  
 X(M)= 0.883713672E 04  
 Y(M)= 0.106543066E 04  
 ST(M)= 0.0  
 RM(M)= 0.0

FY(M)= 0.0  
 V(M)= -0.122472266E 05  
 B(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.150820023E 03  
 RZ(M)= 0.483263245E 02

SEGMENT NUMBER= 3

FX(M)= 0.0  
 FY(M)= 0.0  
 X(M)= 0.883713672E 04  
 Y(M)= 0.106518823E 04  
 ST(M)= 0.0  
 RM(M)= 0.0

FY(M)= 0.0  
 V(M)= -0.107170537E 05  
 B(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.212122421E 03  
 RZ(M)= 0.426410217E 02

SEGMENT NUMBER= 4

FX(M)= 0.0  
 FY(M)= 0.0  
 X(M)= 0.883713672E 04  
 Y(M)= 0.10649759E 04  
 ST(M)= 0.0  
 RM(M)= 0.0

FY(M)= 0.0  
 V(M)= -0.918665625E 04  
 B(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.265261963E 05  
 RZ(M)= 0.369557190E 02

SEGMENT NUMBER= 5

FX(M)= 0.0  
 FY(M)= 0.0  
 X(M)= 0.883713672E 04  
 Y(M)= 0.106479321E 04  
 ST(M)= 0.0  
 RM(M)= 0.0

FY(M)= 0.0  
 V(M)= -0.765595703E 04  
 B(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.310234131E 03  
 RZ(M)= 0.312704163E 02

SEGMENT NUMBER= 6

FX(M)= 0.0  
 FY(M)= 0.0  
 X(M)= 0.883713672E 04  
 Y(M)= 0.106464160E 04  
 ST(M)= 0.0  
 RM(M)= 0.0

FY(M)= 0.0  
 V(M)= -0.612503906E 04  
 B(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.347035156E 03  
 RZ(M)= 0.255851135E 02

SEGMENT NUMBER= 7

FX(M)= 0.0  
 FY(M)= 0.0  
 X(M)= 0.883713672E 04  
 Y(M)= 0.106452031E 04  
 ST(M)= 0.0  
 RM(M)= 0.0

FY(M)= 0.0  
 V(M)= -0.459394922E 04  
 B(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.375661621E 03  
 RZ(M)= 0.198998108E 02

SEGMENT NUMBER= 8

FX(M)= 0.0  
 FY(M)= 0.0  
 X(M)= 0.883713672E 04  
 Y(M)= 0.106442920E 04  
 ST(M)= 0.0  
 RM(M)= 0.0

FY(M)= 0.0  
 V(M)= -0.306272857E 04  
 B(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.396111328E 03  
 RZ(M)= 0.142145081E 02

SEGMENT NUMBER= 9

FX(M)= 0.0  
 FY(M)= 0.0  
 X(M)= 0.883713672E 04  
 Y(M)= 0.106436816E 04  
 ST(M)= 0.0  
 RM(M)= 0.0

FY(M)= 0.0  
 V(M)= -0.153142041E 04  
 B(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.408382568E 03  
 RZ(M)= 0.852920332E 01



SEGMENT NUMBER=10  
 FX(M)= 0.0  
 X(M)= 0.883713672E 04  
 ST(M)= 0.106433813E 04  
 RX(M)= 0.0  
 FY(M)= 0.0  
 Y(M)= -0.688476569E -01  
 R(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.412474121E 03  
 RZ(M)= 0.284390259E 01

SEGMENT NUMBER=11  
 FX(M)= 0.0  
 X(M)= 0.883713672E 04  
 ST(M)= 0.106433813E 04  
 RX(M)= 0.0  
 FY(M)= 0.0  
 Y(M)= 0.153128271E 04  
 R(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.408385742E 03  
 RZ(M)= -0.284140015E 01

SEGMENT NUMBER=12  
 FX(M)= 0.0  
 X(M)= 0.883713672E 04  
 ST(M)= 0.106433813E 04  
 RX(M)= 0.0  
 FY(M)= 0.0  
 Y(M)= 0.306259082E 04  
 R(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.396117920E 03  
 RZ(M)= -0.852670288E 01

SEGMENT NUMBER=13  
 FX(M)= 0.0  
 X(M)= 0.883713672E 04  
 ST(M)= 0.106442496E 04  
 RX(M)= 0.0  
 FY(M)= 0.0  
 Y(M)= 0.459380859E 04  
 R(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.375671387E 03  
 RZ(M)= -0.142120056E 02

SEGMENT NUMBER=14  
 FX(M)= 0.0  
 X(M)= 0.883713672E 04  
 ST(M)= 0.106452022E 04  
 RX(M)= 0.0  
 FY(M)= 0.0  
 Y(M)= 0.612489844E 04  
 R(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.347048096E 03  
 RZ(M)= -0.198973089E 02

SEGMENT NUMBER=15  
 FX(M)= 0.0  
 X(M)= 0.883713672E 04  
 ST(M)= 0.106464160E 04  
 RX(M)= 0.0  
 FY(M)= 0.0  
 Y(M)= 0.765581250E 04  
 R(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.310250488E 03  
 RZ(M)= -0.245826111E 02

SEGMENT NUMBER=16  
 FX(M)= 0.0  
 X(M)= 0.883713672E 04  
 ST(M)= 0.106479321E 04  
 RX(M)= 0.0  
 FY(M)= 0.0  
 Y(M)= 0.918650781E 04  
 R(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.265281738E 03  
 RZ(M)= -0.312679138E 02

SEGMENT NUMBER=17  
 FX(M)= 0.0  
 X(M)= 0.883713672E 04  
 ST(M)= 0.106497559E 04  
 RX(M)= 0.0  
 FY(M)= 0.0  
 Y(M)= 0.107169414E 05  
 R(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.212145737E 03  
 RZ(M)= -0.369532166E 02

SEGMENT NUMBER=18  
 FX(M)= 0.0  
 X(M)= 0.883713672E 04  
 ST(M)= 0.106518823E 04  
 RX(M)= 0.0  
 FY(M)= 0.0  
 Y(M)= 0.122470703E 05  
 R(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.150846939E 03  
 RZ(M)= -0.426385193E 02

SEGMENT NUMBER=19  
 FX(M)= 0.0  
 X(M)= 0.883713672E 04  
 ST(M)= 0.106549066E 04  
 RX(M)= 0.0  
 FY(M)= 0.0  
 Y(M)= 0.137768516E 05  
 R(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= 0.568530273E 01  
 Z(M)= 0.813905334E 02  
 RZ(M)= -0.483238220E 02

SEGMENT NUMBER=20  
 FX(M)= 0.0  
 X(M)= 0.883713672E 04  
 ST(M)= 0.106570337E 04  
 RX(M)= 0.0  
 FY(M)= 0.106433496E 04  
 Y(M)= 0.153082422E 05  
 R(M)= 0.153135669E 04  
 RV(M)= 0.106433496E 04  
 FZ(M)= -0.540091248E 02  
 Z(M)= 0.378234863E 01  
 RZ(M)= -0.540091248E 02

## EQUILIBRIUM POSITION UNDER GRAVITY FORCES

E= 0.185456514 DELTA= 0.187968135E 00 NO. OF ERROR LOOPS= 41

## CABLE NUMBER= 1

## SEGMENT NUMBER= 1

FX(M,N)= 0.0  
FY(M,N)= 0.0  
X(M,N)= -0.176744727E 05  
Y(M,N)= -0.122939644E 05  
TX(M,N)= 0.862446094E 04  
RX(M,N)= 0.862446094E 04

FZ(M,N)= -0.128710937E 01  
Z(M,N)= 0.358253860E 01  
RY(M,N)= 0.0  
RZ(M,N)= 0.876129687E 04

## SEGMENT NUMBER= 2

FX(M,N)= 0.0  
FY(M,N)= 0.0  
X(M,N)= -0.167928906E 05  
Y(M,N)= 0.122946828E 05  
TX(M,N)= 0.862446094E 04  
RX(M,N)= 0.862446094E 04

FZ(M,N)= C.455810547E 00  
Z(M,N)= 0.899281982E 03  
RY(M,N)= 0.0  
RZ(M,N)= 0.876258994E 04

## SEGMENT NUMBER= 3

FX(M,N)= 0.0  
FY(M,N)= 0.0  
X(M,N)= -0.159112852E 05  
Y(M,N)= 0.122945625E 05  
TX(M,N)= 0.862446094E 04  
RX(M,N)= 0.862446094E 04

FZ(M,N)= 0.455810547E 00  
Z(M,N)= 0.179495801E 04  
RY(M,N)= 0.0  
RZ(M,N)= C.876213281E 04

## SEGMENT NUMBER= 4

FX(M,N)= 0.0  
FY(M,N)= 0.0  
X(M,N)= -0.150296562E 05  
Y(M,N)= 0.122942383E 05  
TX(M,N)= 0.862446094E 04  
RX(M,N)= 0.862446094E 04

FZ(M,N)= 0.455810547E 00  
Z(M,N)= 0.269061133E 04  
RY(M,N)= 0.0  
RZ(M,N)= 0.876167969E 04

## SEGMENT NUMBER= 5

FX(M,N)= 0.0  
FY(M,N)= 0.0  
X(M,N)= -0.141480039E 05  
Y(M,N)= 0.122939141E 05  
TX(M,N)= 0.862446094E 04  
RX(M,N)= 0.862446094E 04

FZ(M,N)= 0.455810547E 00  
Z(M,N)= 0.358624219E 04  
RY(M,N)= 0.0  
RZ(M,N)= 0.876122656E 04

## SEGMENT NUMBER= 6

FX(M,N)= 0.0  
FY(M,N)= 0.0  
X(M,N)= -0.132663281E 05  
Y(M,N)= 0.122935937E 05  
TX(M,N)= 0.862446094E 04  
RX(M,N)= 0.862446094E 04

FZ(M,N)= 0.455810547E 00  
Z(M,N)= 0.448184766E 04  
RY(M,N)= 0.0  
RZ(M,N)= 0.876077344E 04

## SEGMENT NUMBER= 7

FX(M,N)= 0.0  
FY(M,N)= 0.0  
X(M,N)= -0.123946289E 05  
Y(M,N)= 0.122932695E 05  
TX(M,N)= 0.862446094E 04  
RX(M,N)= 0.862446094E 04

FZ(M,N)= 0.455810547E 00  
Z(M,N)= 0.537742969E 04  
RY(M,N)= 0.0  
RZ(M,N)= 0.876032031E 04

## SEGMENT NUMBER= 8

FX(M,N)= 0.0  
FY(M,N)= 0.0  
X(M,N)= -0.115029102E 05  
Y(M,N)= 0.122929451E 05  
TX(M,N)= 0.862446094E 04  
RX(M,N)= 0.862446094E 04

FZ(M,N)= 0.455810547E 00  
Z(M,N)= 0.627298280E 04  
RY(M,N)= 0.0  
RZ(M,N)= 0.875986719E 04

## SEGMENT NUMBER= 9

FX(M,N)= 0.0  
FY(M,N)= 0.0  
X(M,N)= -0.106211690E 05  
Y(M,N)= 0.122926250E 05  
TX(M,N)= 0.862446094E 04  
RX(M,N)= 0.862446094E 04

FZ(M,N)= 0.191687012E 03  
Z(M,N)= 0.716892344E 04  
RY(M,N)= 0.0  
RZ(M,N)= 0.875951406E 04

## SEGMENT NUMBER= 10

FX(M,N)= 0.0  
X(M,N)= 0.084164844E 04  
Y(M,N)= 0.121507773E 05  
Z(M,N)= 0.062446094E 04  
FY(M,N)= 0.0  
Y(M,N)= 0.0  
Z(M,N)= 0.250835547E 04  
FZ(M,N)= 0.394067303E 01  
Z(M,N)= 0.093533294E 03  
RZ(M,N)= 0.854773047E 04

SEGMENT NUMBER=11

FXIP(N)= 0.0  
XIP(N)= -0.706172656F 04  
YIP(N)= 0.121540039E 05  
ZIP(N)= 0.062446094E 04  
FYIP(N)= 0.0  
YIP(N)= 0.0  
ZIP(N)= 0.250835083E 04  
FZIP(N)= 0.187290527E 03  
ZIP(N)= 0.107037344E 05  
RZIP(N)= 0.856379297E 04

SEGMENT NUMBER=12

FX(M,N)= 0.0  
XIP(N)= -0.617955859E 04  
YIP(N)= 0.122866914E 05  
ZIP(N)= 0.062446094E 04  
FY(M,N)= 0.0  
YIP(N)= 0.0  
ZIP(N)= 0.125676270E 04  
FZ(M,N)= 0.455810547E 00  
Z(M,N)= 0.115988516E 05  
RZ(M,N)= 0.875108594E 04

SEGMENT NUMBER=13

FX(M,N)= 0.0  
XIP(N)= -0.525736710E 04  
YIP(N)= 0.122866914E 05  
ZIP(N)= 0.062446094E 04  
FY(M,N)= 0.0  
YIP(N)= 0.0  
ZIP(N)= 0.125676147E 04  
FZ(M,N)= 0.455810547E 00  
Z(M,N)= 0.124939453E 05  
RZ(M,N)= 0.875063281E 04

SEGMENT NUMBER=14

FX(M,N)= 0.0  
XIP(N)= -0.441513234E 04  
YIP(N)= 0.122866914E 05  
ZIP(N)= 0.062446094E 04  
FY(M,N)= 0.0  
YIP(N)= 0.0  
ZIP(N)= 0.125676147E 04  
FZ(M,N)= 0.455810547E 00  
Z(M,N)= 0.133890156E 05  
RZ(M,N)= 0.875017949E 04

SEGMENT NUMBER=15

FXIP(N)= 0.0  
XIP(N)= -0.353291797E 04  
YIP(N)= 0.122857272E 05  
ZIP(N)= 0.062446094E 04  
FYIP(N)= 0.0  
YIP(N)= 0.0  
ZIP(N)= 0.125676147E 04  
FZIP(N)= 0.455810547E 00  
ZIP(N)= 0.142840625E 05  
RZIP(N)= 0.874972656E 04

SEGMENT NUMBER=16

FXIP(N)= 0.0  
XIP(N)= -0.265066040E 04  
YIP(N)= 0.122857272E 05  
ZIP(N)= 0.062446094E 04  
FYIP(N)= 0.0  
YIP(N)= 0.0  
ZIP(N)= 0.125676147E 04  
FZIP(N)= 0.455810547E 00  
ZIP(N)= 0.151790859E 05  
RZIP(N)= 0.874927344E 04

SEGMENT NUMBER=17

FXIP(N)= 0.0  
XIP(N)= -0.176837988E 04  
YIP(N)= 0.122857272E 05  
ZIP(N)= 0.062446094E 04  
FYIP(N)= 0.0  
YIP(N)= 0.0  
ZIP(N)= 0.125676147E 04  
FZIP(N)= 0.455810547E 00  
ZIP(N)= 0.160740859E 05  
RZIP(N)= 0.874836719E 04

SEGMENT NUMBER=18

FXIP(N)= 0.0  
XIP(N)= -0.886076172E 03  
YIP(N)= 0.122847539E 05  
ZIP(N)= 0.062446094E 04  
FYIP(N)= 0.0  
YIP(N)= 0.0  
ZIP(N)= 0.125676147E 04  
FZIP(N)= 0.455810547E 00  
ZIP(N)= 0.169690625E 05  
RZIP(N)= 0.874836719E 04

SEGMENT NUMBER=19

FXIP(N)= 0.0  
XIP(N)= -0.374926758E 01  
YIP(N)= 0.122844346E 05  
ZIP(N)= 0.062446094E 04  
FYIP(N)= 0.0  
YIP(N)= 0.0  
ZIP(N)= 0.125676147E 04  
FZIP(N)= 0.128697209E 01  
ZIP(N)= 0.178640154E 05  
RZIP(N)= 0.874791404E 04

SEGMENT NUMBER=20

FXIP(N)= 0.0  
XIP(N)= -0.22022473E 00  
YIP(N)= 0.122839477E 05  
ZIP(N)= 0.062446094E 04  
FYIP(N)= 0.0  
YIP(N)= 0.0  
ZIP(N)= 0.125676147E 04  
FZIP(N)= 0.260954531E 05  
ZIP(N)= 0.178679937E 05  
RZIP(N)= 0.874920312E 04



# EQUILIBRIUM POSITION UNDER ACTING FORCES

E = 0.661418247E-01 DELTA = 0.107968135E 00  
 NO. OF ERROR LOOPS = 24 NO. OF ACCURACY LOOPS = 1  
 CURRENT ANGLE = 0.0

CARLE NUMBER = 1

SEGMENT NUMBER = 1

FX(P,N) = 0.19440875E 02 FZ(M,N) = -0.86016307E 01  
 X(M,N) = -0.17674367E 05 Z(M,N) = 0.368559013E 01  
 TY(P,N) = 0.144670820E 05  
 RX(M,N) = 0.104394727E 05 RZ(M,N) = 0.100156836E 05  
 HORIZ(L,M,N) = 0.105468750E 00 HEIGHT(M,N) = -0.989484787E-01

SEGMENT NUMBER = 2

FX(P,N) = 0.388785095E 02 FZ(M,N) = -0.141364994E 02  
 X(M,N) = -0.167678047E 05 Z(M,N) = 0.875611161E 03  
 TY(P,N) = 0.144590234E 05  
 RX(M,N) = 0.104200352E 05 RZ(M,N) = C.100242852E 05  
 HORIZ(L,M,N) = 0.250859222E 02 HEIGHT(M,N) = -0.236701660E 02

SEGMENT NUMBER = 3

FX(P,N) = 0.389928589E 02 FZ(M,N) = -0.141793261E 02  
 X(M,N) = -0.158634961E 05 Z(M,N) = 0.175006152E 04  
 TY(P,N) = 0.144408555E 05  
 RX(M,N) = 0.103811402E 05 RZ(M,N) = 0.100384219E 05  
 HORIZ(L,M,N) = 0.477890472E 02 HEIGHT(M,N) = -0.448964844E 02

SEGMENT NUMBER = 4

FX(P,N) = 0.391081841E 02 FZ(M,N) = -0.142219142E 02  
 X(M,N) = -0.149614665E 05 Z(M,N) = 0.262684033E 04  
 TY(P,N) = 0.144227305E 05  
 RX(M,N) = 0.103421680E 05 RZ(M,N) = 0.100526016E 05  
 HORIZ(L,M,N) = 0.681952820E 02 HEIGHT(M,N) = -0.637709961E 02

SEGMENT NUMBER = 5

FX(P,N) = 0.392243958E 02 FZ(M,N) = -0.142642326E 02  
 X(M,N) = -0.140617148E 05 Z(M,N) = 0.350595483E 04  
 TY(P,N) = 0.144046523E 05  
 RX(M,N) = 0.103030625E 05 RZ(M,N) = 0.100668242E 05  
 HORIZ(L,M,N) = 0.862890472E 02 HEIGHT(M,N) = -0.802873535E 02

SEGMENT NUMBER = 6

FX(P,N) = 0.393415985E 02 FZ(M,N) = -0.143062744E 02  
 X(M,N) = -0.131642773E 05 Z(M,N) = 0.438741016E 04  
 TY(P,N) = 0.143866172E 05  
 RX(M,N) = 0.102638398E 05 RZ(M,N) = 0.100810898E 05  
 HORIZ(L,M,N) = 0.102050764E 03 HEIGHT(M,N) = -0.944375000E 02

SEGMENT NUMBER = 7

FX(P,N) = 0.394598234E 02 FZ(M,N) = -0.143480864E 02  
 X(M,N) = -0.122691680E 05 Z(M,N) = 0.527121484E 04  
 TY(P,N) = 0.143686250E 05  
 RX(M,N) = 0.102245000E 05 RZ(M,N) = 0.100953984E 05  
 HORIZ(L,M,N) = 0.115460922E 03 HEIGHT(M,N) = -0.106214844E 03

SEGMENT NUMBER = 8

FX(P,N) = 0.395790405E 02 FZ(M,N) = -0.143896045E 02

X(M,N)= 0.11376423E 05  
Y(M,N)= 0.143506836E 05  
Z(M,N)= 0.101850430E 05  
R(M,N)= 0.126507797E 03  
HORIZ(L,M,N)= 0.139144047E 03  
FV(M,N)= 0.20423424E-12  
V(M,N)= 0.12578986E 04  
BL(M,N)= 0.11355098E-12  
RV(M,N)= 0.11355098E-12  
HEIGHT(M,N)= 0.11355098E-12  
RZ(M,N)= 0.101097500E 05

SEGMENT NUMBER= 9

FX(M,N)= 0.651191406E 02  
FY(M,N)= 0.31442332E-13  
FM(M,N)= 0.21099577E-12  
V(M,N)= 0.12578986E 04  
BL(M,N)= 0.11355098E-12  
RV(M,N)= 0.11355098E-12  
HEIGHT(M,N)= 0.11355098E-12  
RZ(M,N)= 0.101241406E 05  
HORIZ(L,M,N)= 0.139144047E 03  
FV(M,N)= 0.270671030E-13  
V(M,N)= 0.21874543E-12  
BL(M,N)= 0.25105637E 04  
RV(M,N)= 0.438958867E-13  
HEIGHT(M,N)= 0.126375000E 03  
RZ(M,N)= 0.995464067E 04

SEGMENT NUMBER= 10

FX(M,N)= 0.789769135E 02  
FY(M,N)= 0.21874543E-12  
FM(M,N)= 0.25105637E 04  
V(M,N)= 0.141671523E 05  
BL(M,N)= 0.100803477E 05  
RV(M,N)= 0.100803477E 05  
HEIGHT(M,N)= 0.141984360E 03  
RZ(M,N)= 0.139140610E 03  
FV(M,N)= 0.105087820E-13  
V(M,N)= 0.221764718E-12  
BL(M,N)= 0.251052075E 04  
RV(M,N)= 0.16828782E-13  
HEIGHT(M,N)= 0.120441406E 03  
RZ(M,N)= 0.998028906E 04

SEGMENT NUMBER= 11

FX(M,N)= 0.57936401E 02  
FY(M,N)= 0.221764718E-12  
FM(M,N)= 0.251052075E 04  
V(M,N)= 0.141291719E 05  
BL(M,N)= 0.100013711E 05  
RV(M,N)= 0.100013711E 05  
HEIGHT(M,N)= 0.139140610E 03  
RZ(M,N)= 0.135632797E 03  
FV(M,N)= 0.173420827E-14  
V(M,N)= 0.22267902E-12  
BL(M,N)= 0.175783789E 04  
RV(M,N)= 0.63200081E-14  
HEIGHT(M,N)= 0.11549644E 03  
RZ(M,N)= 0.101899492E 05

SEGMENT NUMBER= 12

FX(M,N)= 0.402956696E 02  
FY(M,N)= 0.22267902E-12  
FM(M,N)= 0.175783789E 04  
V(M,N)= 0.142404102E 05  
BL(M,N)= 0.99457812E 04  
RV(M,N)= 0.99457812E 04  
HEIGHT(M,N)= 0.135632797E 03  
RZ(M,N)= 0.129617172E 03  
FV(M,N)= 0.309717322E-14  
V(M,N)= 0.22267902E-12  
BL(M,N)= 0.175783789E 04  
RV(M,N)= 0.63200081E-14  
HEIGHT(M,N)= 0.11549644E 03  
RZ(M,N)= 0.101899492E 05

SEGMENT NUMBER= 13

FX(M,N)= 0.404218292E 02  
FY(M,N)= 0.22267902E-12  
FM(M,N)= 0.175783789E 04  
V(M,N)= 0.142227852E 05  
BL(M,N)= 0.990228516E 04  
RV(M,N)= 0.990228516E 04  
HEIGHT(M,N)= 0.129617172E 03  
RZ(M,N)= 0.129617172E 03  
FV(M,N)= 0.173420827E-14  
V(M,N)= 0.22267902E-12  
BL(M,N)= 0.175783789E 04  
RV(M,N)= 0.63200081E-14  
HEIGHT(M,N)= 0.11549644E 03  
RZ(M,N)= 0.101899492E 05

SEGMENT NUMBER= 14

FX(M,N)= 0.405489960E 02  
FY(M,N)= 0.22267902E-12  
FM(M,N)= 0.175783789E 04  
V(M,N)= 0.142227852E 05  
BL(M,N)= 0.990228516E 04  
RV(M,N)= 0.990228516E 04  
HEIGHT(M,N)= 0.129617172E 03  
RZ(M,N)= 0.129617172E 03  
FV(M,N)= 0.884906755E-15  
V(M,N)= 0.222739687E-12  
BL(M,N)= 0.125781783E 04  
RV(M,N)= 0.14886222E-14  
HEIGHT(M,N)= 0.983242180E 02  
RZ(M,N)= 0.102192461E 05

SEGMENT NUMBER= 15

FX(M,N)= 0.406771240E 02  
FY(M,N)= 0.22267902E-12  
FM(M,N)= 0.175783789E 04  
V(M,N)= 0.142227852E 05  
BL(M,N)= 0.982632031E 04  
RV(M,N)= 0.982632031E 04  
HEIGHT(M,N)= 0.10995346E 03  
RZ(M,N)= 0.10995346E 03  
FV(M,N)= 0.398635304E-15  
V(M,N)= 0.222739687E-12  
BL(M,N)= 0.125781783E 04  
RV(M,N)= 0.14886222E-14  
HEIGHT(M,N)= 0.983242180E 02  
RZ(M,N)= 0.102192461E 05

SEGMENT NUMBER= 16

FX(M,N)= 0.408062744E 02  
FY(M,N)= 0.22267902E-12  
FM(M,N)= 0.175783789E 04  
V(M,N)= 0.142227852E 05  
BL(M,N)= 0.978564453E 04  
RV(M,N)= 0.978564453E 04  
HEIGHT(M,N)= 0.963466644E 02  
RZ(M,N)= 0.963466644E 02  
FV(M,N)= 0.151004256E-15  
V(M,N)= 0.222811353E-12  
BL(M,N)= 0.125779858E 04  
RV(M,N)= 0.205080256E-15  
HEIGHT(M,N)= 0.713945313E 02  
RZ(M,N)= 0.102486992E 05

SEGMENT NUMBER= 17

FX(M,N)= 0.409364319E 02  
FY(M,N)= 0.22267902E-12  
FM(M,N)= 0.175783789E 04  
V(M,N)= 0.142227852E 05  
BL(M,N)= 0.974493984E 04  
RV(M,N)= 0.974493984E 04  
HEIGHT(M,N)= 0.940759995E-16  
RZ(M,N)= 0.940759995E-16  
FV(M,N)= 0.444461481E-16  
V(M,N)= 0.222811353E-12  
BL(M,N)= 0.125779858E 04  
RV(M,N)= 0.205080256E-15  
HEIGHT(M,N)= 0.713945313E 02  
RZ(M,N)= 0.102486992E 05



MORZL(N)= 0.80111801E 02      HEIGHT(N)=0.542578125E 02  
 SEGMENT NUMBER=18  
 FY(N)= 0.41067738E 02      FY(N)=0.88178644E-17      FZ(N)=0.148553724E 02  
 X(N)=0.824808396E 03      Y(N)=0.222816936E-12      Z(N)= 0.169343984E 05  
 TY(N)= 0.141393867E 05      BL(N)= 0.125777959E 04  
 RX(N)= 0.070390625E 04      RY(N)=0.962985554E-17      RZ(N)= 0.102783008E 05  
 MORZL(N)= 0.612697601E 02      HEIGHT(N)=0.344440425E 02  
 SEGMENT NUMBER=19  
 FY(N)= 0.206017761E 02      FY(N)=0.811991095E-18      FZ(N)=0.898250580E 01  
 X(N)= 0.360503371E 02      Y(N)=0.222816991E-12      Z(N)= 0.178514062E 05  
 TY(N)= 0.141180547E 05      BL(N)= 0.125777002E 04  
 RX(N)= 0.96428354E 04      RY(N)=0.811991095E-18      RZ(N)= 0.102931562E 05  
 MORZL(N)= 0.397997894E 02      HEIGHT(N)=0.12609375CE 02  
 SEGMENT NUMBER=20  
 FY(N)= 0.531686279E 03      FY(N)= 0.0      FZ(N)= 0.260954648E 05  
 X(N)= 0.394884338E 02      Y(N)=0.222816991E-12      Z(N)= 0.178550781E 05  
 TY(N)= 0.141105195E 05      BL(N)= 0.503106594E 01  
 RX(N)= 0.964223828E 04      RY(N)= 0.0      RZ(N)= 0.103021406E 05  
 MORZL(N)= 0.397086334E 02      HEIGHT(N)=0.125156250E 02  
 CARLE NUMBER= 2  
 SEGMENT NUMBER= 1  
 FY(N)= 0.28832732E 02      FY(N)= 0.465512848E 01      FZ(N)= 0.411708439E 01  
 X(N)= 0.415003204E 02      Y(N)=0.300659466E 01      Z(N)= 0.178515898E 05  
 TY(N)= 0.113775644E 05      BL(N)= 0.502504826E 01  
 RX(N)= 0.455527734E 04      RY(N)=0.680744531E 04      RZ(N)=0.789666016E 04  
 MORZL(N)= 0.399482880E 02      HEIGHT(N)=0.1243750CCE 02  
 SEGMENT NUMBER= 2  
 FY(N)= 0.576410980E 02      FY(N)= 0.923836134E 01      FZ(N)= 0.111710978E 02  
 X(N)= 0.541546875E 03      Y(N)=0.755554688E 03      Z(N)= 0.169787734E 05  
 TY(N)= 0.113717031E 05      BL(N)= 0.125625364E 04  
 RX(N)= 0.452644531E 04      RY(N)=0.68213984E 04      RZ(N)=0.790077344E 04  
 MORZL(N)= 0.980391541E 02      HEIGHT(N)= 0.583203125E 01  
 SEGMENT NUMBER= 3  
 FY(N)= 0.57861756E 02      FY(N)= 0.916803265E 01      FZ(N)= 0.110901337E 02  
 X(N)= 0.103563672E 04      Y(N)=0.150975073E 04      Z(N)= 0.161039922E 05  
 TY(N)= 0.113622031E 05      BL(N)= 0.125625364E 04  
 RX(N)= 0.446880469E 04      RY(N)=0.68213984E 04      RZ(N)=0.791194531E 04  
 MORZL(N)= 0.150552734E 03      HEIGHT(N)= 0.221801563E 02  
 SEGMENT NUMBER= 4  
 FY(N)= 0.560816345E 02      FY(N)= 0.909574318E 01      FZ(N)= 0.110069008E 02  
 X(N)= 0.152372949E 04      Y(N)=0.226558008E 04      Z(N)= 0.152272656E 05  
 TY(N)= 0.113528320E 05      BL(N)= 0.125625754E 04  
 RX(N)= 0.441094531E 04      RY(N)=0.683050781E 04      RZ(N)=0.792303906E 04  
 MORZL(N)= 0.196960144E 03      HEIGHT(N)= 0.365625000E 02  
 SEGMENT NUMBER= 5  
 FY(N)= 0.583007355E 02      FY(N)= 0.902149109E 01      FZ(N)= 0.109213705E 02  
 X(N)= 0.200578711E 04      Y(N)=0.302303125E 04      Z(N)= 0.143486094E 05  
 TY(N)= 0.113455742E 05      BL(N)= 0.125624292E 04  
 RX(N)= 0.435286719E 04      RY(N)=0.683960547E 04      RZ(N)=0.793404687E 04  
 MORZL(N)= 0.237160553E 03      HEIGHT(N)= 0.490625000E 02  
 SEGMENT NUMBER= 6  
 FY(N)= 0.585189C56E 02      FY(N)= 0.894524097E 01      FZ(N)= 0.108335400E 02  
 X(N)= 0.248176978E 04      Y(N)=0.378708960E 04      Z(N)= 0.134680391E 05  
 TY(N)= 0.113344414E 05      BL(N)= 0.125623804E 04  
 RX(N)= 0.429457031E 04      RY(N)=0.684862891E 04      RZ(N)=0.794494875E 04



# EQUILIBRIUM POSITION UNDER ACTING FORCES

E = 0.15220163E 03 DELTA = 0.15274439E 03  
 NO. OF ERROR LOOPS = 124 NO. OF ACCURACY LOOPS = 15  
 CURRENT ANGLE = 0.0

CIRCULAR DISTANCE BETWEEN END POINTS = 0.306108203E 05

## SEGMENT NUMBER = 1

FX(M) = 0.70177034E 02 FZ(M) = 0.57358659E 01 FZ(M) = 0.56340446E 01  
 X(M) = 0.40180742E 04 Y(M) = -0.13781582E 05 Z(M) = 0.17264221E 02  
 ST(M) = 0.57240781E 04 R(M) = 0.15344965E 04  
 RI(M) = 0.67159227E 03 RY(M) = 0.56842968E 04 RZ(M) = 0.538140564E 02  
 MORI(ZL(M) = 0.18099520E 03 HEIGHT(M) = 0.64095704E 02

## SEGMENT NUMBER = 2

FX(M) = 0.703657074E 02 FZ(M) = -0.51111154E 01 FZ(M) = 0.56446294E 01  
 X(M) = 0.91793554E 04 Y(M) = -0.12255640E 05 Z(M) = 0.30189028E 02  
 ST(M) = 0.57219296E 04 R(M) = 0.15344951E 04  
 RI(M) = 0.60140502E 03 RY(M) = 0.56900351E 04 RZ(M) = 0.48180023E 02  
 MORI(ZL(M) = 0.34232202E 03 HEIGHT(M) = 0.12063499E 03

## SEGMENT NUMBER = 3

FX(M) = 0.705310059E 02 FZ(M) = -0.44818038E 01 FZ(M) = 0.56540317E 01  
 X(M) = 0.53218125E 04 Y(M) = -0.10727812E 05 Z(M) = 0.41595901E 02  
 ST(M) = 0.57200078E 04 R(M) = 0.15344951E 04  
 RI(M) = 0.53103952E 03 RY(M) = 0.56951484E 04 RZ(M) = 0.42535404E 02  
 MORI(ZL(M) = 0.48479418E 03 HEIGHT(M) = 0.17052652E 03

## SEGMENT NUMBER = 4

FX(M) = 0.70472760E 02 FZ(M) = -0.38444611E 01 FZ(M) = 0.56622276E 01  
 X(M) = 0.44538672E 04 Y(M) = -0.91983320E 04 Z(M) = 0.51492889E 02  
 ST(M) = 0.57183242E 04 R(M) = 0.15344936E 04  
 RI(M) = 0.44050894E 03 RY(M) = 0.56996328E 04 RZ(M) = 0.36881378E 02  
 MORI(ZL(M) = 0.60836181E 03 HEIGHT(M) = 0.21376907E 03

## SEGMENT NUMBER = 5

FX(M) = 0.70796573E 02 FZ(M) = -0.32116947E 01 FZ(M) = 0.56691970E 01  
 X(M) = 0.95500234E 04 Y(M) = -0.76674296E 04 Z(M) = 0.59872558E 02  
 ST(M) = 0.57168710E 04 R(M) = 0.15344921E 04  
 RI(M) = 0.38483593E 03 RY(M) = 0.57034843E 04 RZ(M) = 0.31219162E 02  
 MORI(ZL(M) = 0.71297976E 03 HEIGHT(M) = 0.25036157E 03

## SEGMENT NUMBER = 6

FX(M) = 0.70884124E 02 FZ(M) = -0.25721225E 01 FZ(M) = 0.56749219E 01  
 X(M) = 0.96356757E 04 Y(M) = -0.61353437E 04 Z(M) = 0.66731979E 02  
 ST(M) = 0.57156646E 04 R(M) = 0.15344921E 04  
 RI(M) = 0.31904541E 03 RY(M) = 0.57066922E 04 RZ(M) = 0.25549072E 02  
 MORI(ZL(M) = 0.79860546E 03 HEIGHT(M) = 0.28030297E 03

## SEGMENT NUMBER = 7

FX(M) = 0.70953506E 02 FZ(M) = -0.19303054E 01 FZ(M) = 0.56793880E 01  
 X(M) = 0.97023085E 04 Y(M) = -0.44023085E 04 Z(M) = 0.72068756E 02  
 ST(M) = 0.57146931E 04 R(M) = 0.15344907E 04  
 RI(M) = 0.24816133E 03 RY(M) = 0.57092734E 04 RZ(M) = 0.19879061E 02  
 MORI(ZL(M) = 0.86521219E 03 HEIGHT(M) = 0.30399277E 03

## SEGMENT NUMBER = 8

FX(M) = 0.70998937E 02 FZ(M) = -0.12867307E 01 FZ(M) = 0.56825847E 01  
 RI(M) = 0.97498949E 04 Y(M) = -0.39463991E 05 Z(M) = 0.75901012E 02





SEGMENT NUMBER=18  
 FX(M)= 0.701047058E 02  
 X(M)= 0.917870703E 04  
 ST(M)= 0.57199609E 04  
 RX(M)= -C.530747314E 03  
 HCRIZL(M)= 0.341683105E 03  
 FV(M)= 0.510740280E 01  
 V(M)= 0.122558555E 05  
 B(M)= 0.153449512E 04  
 RV(M)= 0.569516406E 04  
 HEIGHT(M)= -C.120721954E 03  
 FZ(M)= 0.564463425E 01  
 Z(M)= 0.301249847E 02  
 RZ(M)= -0.425612935E 02

SEGMENT NUMBER=19  
 FX(M)= 0.698954218E 02  
 X(M)= 0.901757031E 04  
 ST(M)= 0.572189453E 04  
 RX(M)= -C.600652051E 03  
 HCRIZL(M)= 0.180501770E 03  
 FV(M)= 0.572977666E 01  
 V(M)= 0.137818175E 05  
 B(M)= 0.153449512E 04  
 RV(M)= 0.569005859E 04  
 HEIGHT(M)= -0.641939392E 02  
 FZ(M)= 0.563407135E 01  
 Z(M)= 0.171965942E 02  
 RZ(M)= -0.482079315E 02

SEGMENT NUMBER=20  
 FX(M)= 0.670737549E 03  
 X(M)= 0.883775781E 04  
 ST(M)= 0.572401953E 04  
 RX(M)= -0.670737549E 03  
 HCRIZL(M)= 0.845863223E 00  
 FV(M)= 0.568433203E 04  
 V(M)= 0.153056680E 05  
 B(M)= 0.153449658E 04  
 RV(M)= 0.568433203E 04  
 HEIGHT(M)= -0.101573248E 01  
 FZ(M)= -0.538420105E 02  
 Z(M)= 0.276261616E 01  
 RZ(M)= -0.538420105E 02



## APPENDIX II

### SPECIAL FEATURES FOR USING THE COMPUTER PROGRAM

The Computer Program that has been described in Appendix I can be used by using appropriate data cards. Because of the truncation errors made by the computer it is necessary to choose the appropriate cutoff values that will suggest that the method has been used satisfactorily to simulate the cable system. A brief description of these cutoff values is made in the following paragraphs:

A. Cutoff Value to Define the Acceptable Completion of Imaginary Reaction Routine

As described earlier in Chapter I, the equilibrium configuration of the array system is obtained if  $E$ , the measure of error between the end coordinates, as obtained by system of forces at hand and the correct ones as specified, is zero. Theoretically, the iteration would continue until  $E$  is identically zero i.e., until the equilibrium configuration (under the constant applied forces) is obtained exactly. However, because of computer roundoff errors, it is not possible to achieve this and as such a cutoff value, that would define the acceptable completion of the imaginary reaction routine, has to be defined.

If this cutoff value is denoted by  $COMPE$ , then the imaginary reaction routine is considered to have given the satisfactory equilibrium configuration of the cable system when

$$E \leq COMPE \quad (1,II)$$

That is, when

$X_{M(n),n}$ :  $Y_{M(n),n}$ :  $Z_{M(n),n}$  for  $n = 2$  and  $3$  for the main arrays and  $X_{MN}$ :  $Y_{MN}$ :  $Z_{MN}$  for the tie leg array, are all within  $COMPE$  from their true anchor values.

B. Cutoff Value to Define the Acceptable Completion of the Successive Approximation Iterations

As described in Chapter I COMPD denotes the fixed accuracy value and is used to suggest the acceptable completion of the successive approximation routine. That is, the equilibrium coordinates of any cable station for two successive iterations are compared. If the coordinates differ by less than COMPD, the iteration is considered satisfied.

However, introduction of COMPE as a cutoff value also introduces an error, again within  $\sqrt{\text{COMPE}}$  from the actual equilibrium coordinates, into the coordinates calculated for every cable station. With the result, it is important that the cutoff value COMPD for the successive approximation iteration be chosen outside the value of this inherent error.

A safe minimum value for  $\text{COMPD}^{(9)}$  is

$$\text{COMPD} = 10 \sqrt{\text{COMPE}}$$

C. Cutoff Value to Define the Acceptable Completion of the Force Balance Condition

The force balance condition, as described in Chapter IV, is said to have been satisfied if

$$\text{FFX} + \text{RX1} = 0$$

$$\text{FFY} + \text{RY1} = 0$$

$$\text{FFZ} + \text{RZ1} = 0$$

where FFY and RY1 are defined by equations (1,4). FFX, FFZ and RX1, RZ1 are similar forces in the X and Z direction respectively.

However, because of computer roundoff error some cutoff value is defined which would suggest that the force balance condition has been satisfied to an acceptable degree. This value is defined by TIECOM.

TIECOM can be made very small, since binary search routine will make the values of FFY and RY1 converge to an acceptable positive number. An acceptable value for TIECOM could be 1 lb.

#### D. PRECISION FOCUS

This convergence concept was suggested by G. H. Savage<sup>(13)</sup> and is used in conjunction with the  $\delta$  method of convergence discussed in Chapter I. It utilizes the past performance of the force and displacement to bring the cable array to the required coordinates. Referring to Figure 17, the end of the cable is released from point K' and imaginary reactions are applied at the free end. For each iteration, depending upon the position of the K end, additive forces given by equation (20,1) are added to the imaginary reactions. After a certain number of iterations, it is possible to bring the K end to its X and Z coordinates within limits of COMPE. Normally the above process would be applied until y coordinate is also within the limits of COMPE. However, it has been found that this situation is not always possible to achieve. The positive number  $\delta$  often becomes too small to make any significant difference in the additive force given by equation (20,1), the E never becomes less than COMPE and, therefore, equilibrium configuration is not obtained. To cope with this problem, after x and z coordinates have been obtained within the limits of COMPE, the Precision Focus convergence approach is applied. To demonstrate this method, Figure 17 represents the plot between force FY, which is the end reaction, and the Y coordinate. Pt. H represents a point of intersection, the point which represents the exact y coordinate to which the k end of the cable should strive to reach.





Since the above approach utilizes the past performance and takes into account the slope of the curve, this method seems to avoid the problem posed by the positive number  $\delta$  getting too small.

Close to point H, the curve gets flat with the result that it is possible to get into a position wherein points P and M are same i.e.,  $PQ - MN = 0$ . When this happens the control is reverted back to the previous convergence concept. This concept is utilized until x and z coordinates are within the limits of COMPE, at which point control is passed over to Precision Focus again. This process is then repeated until  $E \leq \text{COMPE}$  at which point equilibrium configuration is obtained.

#### E. ASSUMPTIONS USED IN THE COMPUTER MODEL\*

In this section a discussion is made of the assumptions made in the computer model. These assumptions are made only for the purpose of convenience and are not necessary from a theoretical point of view.

1. Subroutine VPROFL deals with the current profile. In the program, a profile assuming a series of straight line segments of arbitrary length and scope has been used. However, the analysis developed has been unrestricted. The analysis requires evaluation of the integral as defined by equations (29,3). Thus, any velocity profile could be used as long as the above integral could be evaluated.  
  
However, if current profile consists of series of straight line segments, as in our case, this integral can be evaluated exactly\*\*. As such, this profile was chosen to write the computer program.
2. On all runs made with the computer program as given in Appendix I, a uniform current profile was used. Current velocity of 0.6739 per sec. was assumed all the way from the bottom ( $z = 0$ ) to the surface. This

\*The reader who desires to use the computer program using a different data set than listed in Appendix I is referred to Sub-Sections 6 and 7 of this Section.

\*\*The evaluation is performed in Appendix II, Skop and Kaplan, "Static Configuration of a Tri-Moored, Subsurface Buoy."



velocity profile, as suggested earlier, was broken up into series of straight line segments. The program is written to take into consideration any velocity profile, but a constant profile was used in this study to provide easy comparison of hydrodynamic forces at all points on the array system.

3. Only one angle of attack of current was used to study the hydrodynamic behavior of the structure. The facility to have a different angle of attack exists in the program.

It was observed from various computer runs made with the computer program as listed in Reference 9, and modified to suit the facility at the University of New Hampshire, that the worst displacements of the Subsurface Buoy were encountered with the angle of attack of the current as zero degrees ( $\beta=0$ ). As such, for the analysis of the present array system, current with only this angle of attack was used.

4. As described in Chapter IV, a Binary Search Routine was suggested to be used to satisfy the force balance condition in a minimum number of subroutine iterations. Forces in the y - coordinate directions posed the majority of problems. As such, this routine was used only in the y - direction. Should continued use of this program indicate difficulties in the X or Z coordinate directions, then the Binary Search Routine may need to be applied there too.
5. The results that are discussed in Chapter V were obtained by using some of the data listed in a report by R. G. Paquette entitled "Seaspider Hydrodynamics," dated 24 April 1969. This uses a surface buoy along with the subsurface buoy in accordance with the lumped parameter representation. This, however, is not necessary and any other data set can be used.
6. The length of the tie leg array and length of each segment of the tie leg array were determined by the program internally. This also was done only for



simple convenience. If the length of the tie leg is to be given, then a change in the program would have to be made accordingly.

7. The tie leg array is used as a neutrally buoyant array. However, since it is not possible to have this in practice, it was decided to make the array 1% positively buoyant. This was done by using floating objects, with their weights in water as a reference. From this the weight of cable/ft. in water was determined internally by the program. This situation can very easily be reversed. Also, since the length of each segment was determined internally, it was also decided to space the floating objects equally in the segments by the program internally. This is completely for the purpose of convenience.

#### F. OTHER CONSIDERATIONS

Besides the above cutoff values there are special cases in which the program may never obtain the equilibrium configuration i.e., E may never obtain the value of COMPE. There can be one of the two possible reasons for this to happen.

1. The convergence factor  $\delta$ , as discussed in Chapter 1, approaches zero as E approaches zero. Consequently, for a very small value of  $\delta$  no change will occur in the applied imaginary reactions as a result of the significant figure limitations of the computer. When this happens COMPE is too small. This problem can normally be alleviated by either using Precision Focus or increasing the value of COMPE.
2. One of the cable segments has developed zero tension or has gone slack. In this case the array is statically unstable. This condition also results from no change in imaginary reaction. The computer program for the tri-moored array has been equipped to

give out the error printout for the benefit of the user.

In Figure 18, a logical diagram, showing briefly step-by-step description of how the computer program proceeds to determine the equilibrium configuration of the cable system, is presented.

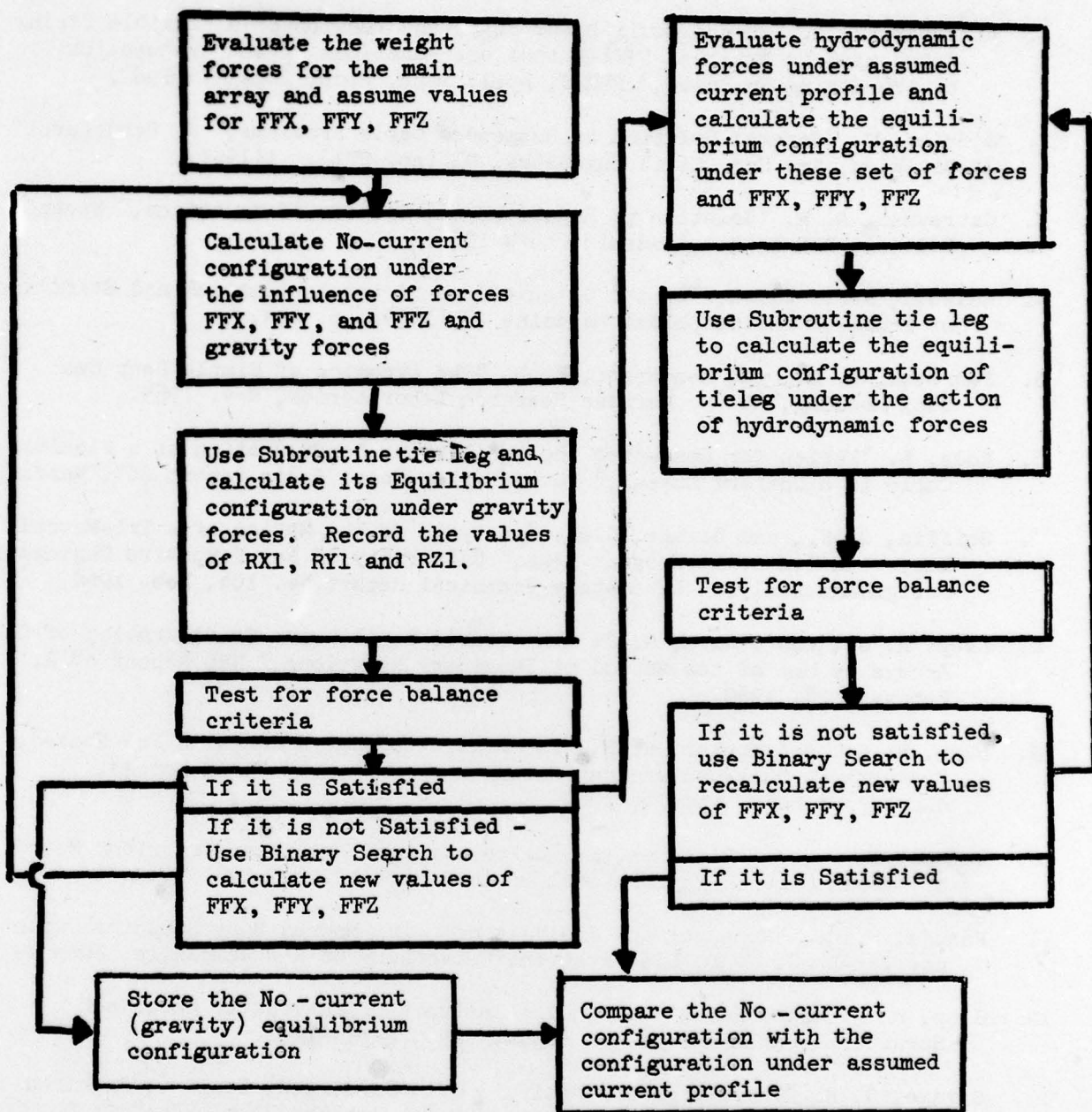


Figure 18: Logic Diagram Showing Step-by-Step Description of Computer Program.



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